PRINCIPLES OF INERTIAL GUIDANCE

By C. S. Draper*, W. Wrigley† and R. B. Woodbury‡

INTRODUCTION

Guidance is the art and science of causing objects to follow desired paths with respect to designated reference points. The basic problems are those of relative motion geometry in which controlled points are made to move into selected regions associated with the reference points. Various guidance methods are possible and differ among themselves in the co-ordinate systems that are used in solving the geometrical problems involved. A very simple guidance situation is illustrated by Fig. 1a, in which a man in a canoe has a direct line of sight to his destination. The man requires only one set of co-ordinates, although two sets are immediately obvious. He may use co-ordinates fixed to the canoe, so that guidance consists in merely keeping the bow of the canoe pointed toward his destination. He may also use Earth co-ordinates and imagine a line fixed to the Earth and passing through the destination. In this case, guidance is the process of keeping the path of the canoe on the selected line to the destination.

The direct-line-of-sight guidance of Fig. 1a is of no use when the destination is not visible for any reason. When this no-visibility condition exists, it is common practice to introduce an auxiliary reference co-ordinate system with a known relationship to an Earth co-ordinate system and in which it is possible to locate the *present position* of the guided vehicle. Figure 1b illustrates this situation when the reference co-ordinates are established by a set of radio stations fixed to the Earth in the fashion used for Loran and Shoran systems. Equipment carried in the vehicle and the radio-wave pattern set up by the radio stations make it possible to determine present position in the reference co-ordinates. The transfer to Earth co-ordinates is easily made from a knowledge of the station locations on the Earth's surface.

Figure 1c illustrates the essential features of the guidance problem when present position is known. The situation is based on a selection of the path desired for the motion of the controlled vehicle. This path is established

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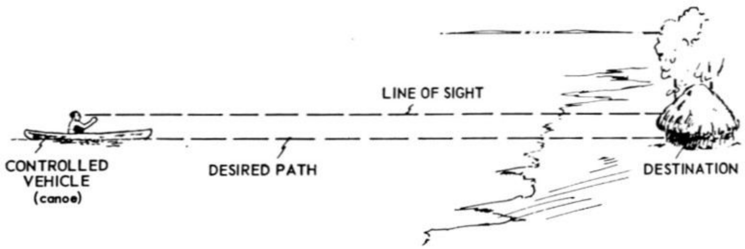
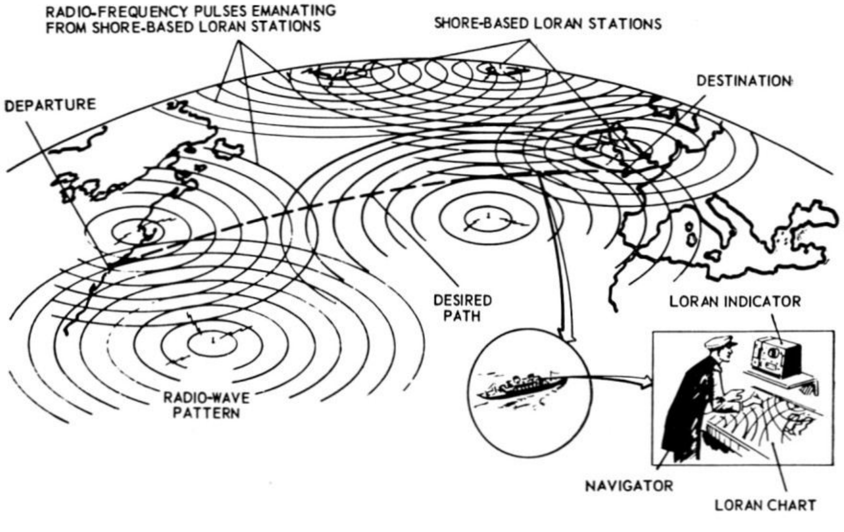
from considerations beyond those of the guidance process that is being considered here. For example, the objective of guidance might be to cause an aircraft to fly from New York to London along a great-circle course. The great circle would become the desired path. If, in addition to merely reaching London, it is intended to have the aircraft reach given positions at particular times according to a *program*, the desired path



LINE OF SIGHT GIVES DIRECT INFORMATION ON RELATIVE POSITION BETWEEN CONTROLLED VEHICLE AND DESTINATION.

SINGLE COORDINATE SPACE FIXED TO CONTROLLED VEHICLE IS SUFFICIENT.

a) Destination directly visible from controlled vehicle



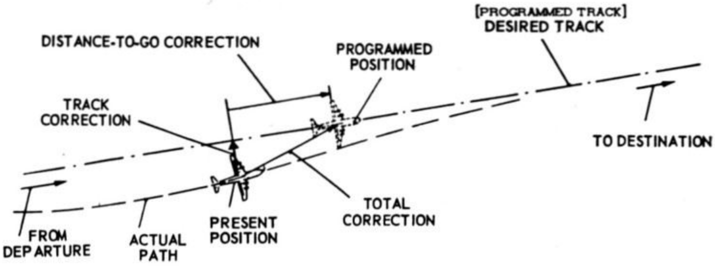
DEPARTURE AND DESTINATION ARE KNOWN IN EARTH COORDINATES (LATITUDE AND LONGITUDE). DESIRED TRACK IS ESTABLISHED FROM KNOWLEDGE OF DEPARTURE AND DESTINATION IN EARTH COORDINATES.

SHORE-BASED STATIONS AND EQUIPMENT ABOARD SHIP COUPLED BY RADIO-WAVE PATTERN GIVE LOCATION OF SHIP IN LORAN COORDINATES (HYPERBOLIC PLOTS BETWEEN LORAN STATIONS). RELATIONSHIP OF LORAN COORDINATES WITH RESPECT TO EARTH COORDINATES ARE AVAILABLE IN FORM OF LORAN CHARTS.

USING KNOWN RELATIONSHIP BETWEEN LORAN COORDINATES AND EARTH COORDINATES, SHIP IS LOCATED IN EARTH COORDINATES TO GIVE PRESENT POSITION.

b) Earth coordinates and a typical reference coordinate system

Fig. 1. Guidance in terrestrial coordinates when a clear line of sight exists or a direct radiation connection is available.



PRESENT POSITION WITH RESPECT TO DESIRED TRACK (THE PROGRAMMED TRACK) GIVES THE TRACK CORRECTION, WHICH IS THE CHANGE IN PRESENT POSITION REQUIRED TO BRING THE VEHICLE TO THE PROGRAMMED TRACK.

PRESENT POSITION WITH RESPECT TO THE PROGRAMMED POSITION (THE POSITION PLANNED FOR THE VEHICLE TO OCCUPY AT A GIVEN INSTANT) GIVES THE DISTANCE-TO-GO CORRECTION, WHICH IS THE CHANGE IN PRESENT POSITION REQUIRED TO BRING THE VEHICLE FROM ITS PRESENT POSITION AT A GIVEN INSTANT TO THE PROGRAMMED POSITION FOR THE SAME INSTANT.

c) Guidance corrections

Fig. 1. Guidance in terrestrial coordinates when a clear line of sight exists or a direct radiation connection is available.

would become a *programmed path*. Figure 1c is an illustrative diagram showing typical relationships among the programmed path, the actual path, and the *corrections* in position required to change the present position so that it becomes identical with the programmed position. The *track correction* is the change in the direction at right angles to the programmed track required to bring the present position to the programmed track. The *distance-to-go correction* is the change in position along the programmed track that would be necessary to make the present position identical with the programmed position.

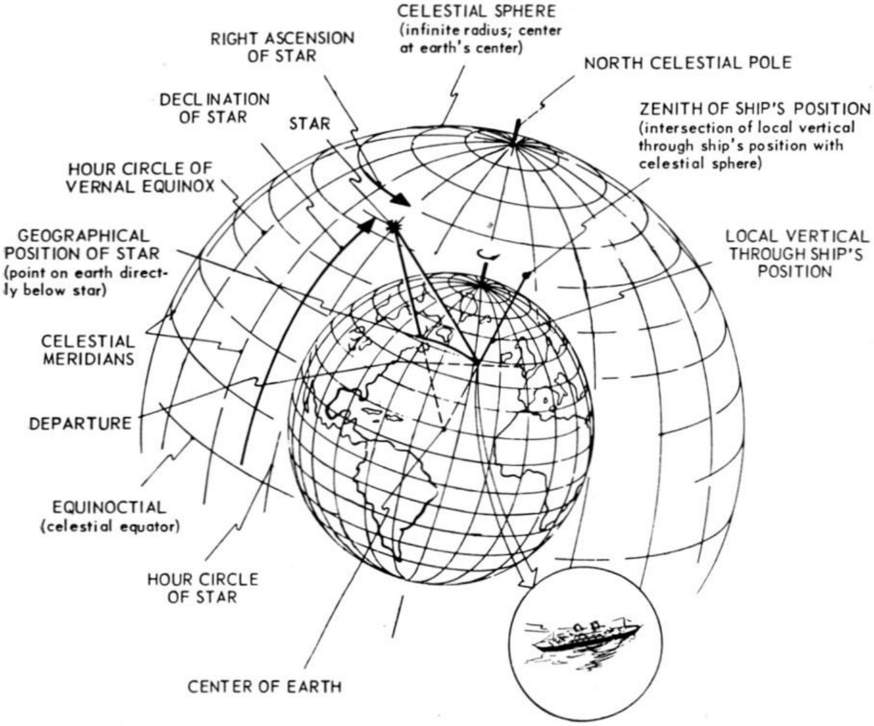
Guidance for a vehicle is the process of changing the vehicle motion so that the corrections illustrated in Fig. 1c are changed toward smaller magnitudes. Ideal magnitudes for these corrections are zero. Any satisfactory guidance system must keep the corrections within acceptable tolerance limits.

The purpose of this paper is to describe the background theory for guidance systems that are not based on the use of lines of sight or radiation contacts, but depend instead on the inertial properties of matter. Systems of this kind have the advantages of being self-contained, so that they do not require co-operating ground equipment and operate without interference from optical, radio, or radar radiation or from weather conditions. The paper outlines the physical principles available for inertial systems and calls attention to the essential problems involved by means of an illustrative system.

AUXILIARY CO-ORDINATES FOR GUIDANCE

Line-of-sight and radiation-connection contacts between reference points and the vehicle to be guided are very often not available. In some situations, visibility to nearby points is eliminated by fog, clouds, rain, or

WHEN DEPARTURE AND DESTINATION ARE BEYOND LINE-OF-SIGHT CONTACT OR DIRECT RADIATION CONNECTION IS NOT POSSIBLE, CELESTIAL REFERENCE COORDINATES MAY BE USED FOR GUIDANCE.



DEPARTURE AND DESTINATION ARE KNOWN IN EARTH COORDINATES AND ESTABLISH THE PROGRAMMED TRACK IN EARTH COORDINATES.

CELESTIAL SPACE, ESTABLISHED BY THE FIXED STARS, MAY BE USED TO PROVIDE REFERENCE COORDINATES FOR GUIDANCE.

AT ANY GIVEN INSTANT, AS ESTABLISHED BY A CHRONOMETER OR SOME OTHER ACCURATE TIME-INDICATING DEVICE, THE POSITION OF THE EARTH COORDINATE SYSTEM WITH RESPECT TO THE CELESTIAL-SPACE COORDINATE SYSTEM IS KNOWN FROM ALMANAC INFORMATION.

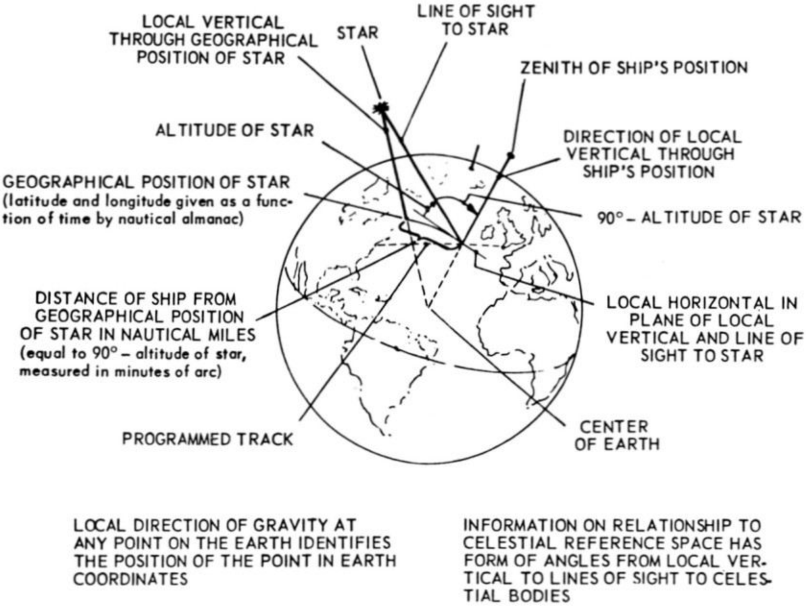
AT A PARTICULAR INSTANT, THE PRESENT POSITION MAY BE ESTABLISHED IN CELESTIAL SPACE BY LINES OF SIGHT TO PROPERLY CHOSEN STARS.

THIS PRESENT POSITION IN CELESTIAL REFERENCE COORDINATES MAY BE TRANSFERRED TO EARTH COORDINATES BY THE USE OF ACCURATE TIME AND INFORMATION ON THE RELATIONSHIP BETWEEN CELESTIAL COORDINATES AND EARTH COORDINATES.

a) Present position in earth coordinates by transfer from celestial reference coordinates

Fig. 2. Guidance by use of celestial reference coordinates when destination is beyond line-of-sight contact or direct radiation connection.

REF. DUTTON, B. J., "NAVIGATION AND NAUTICAL ASTRONOMY."



b) Angles between celestial-body lines of sight and local direction of gravity gives present position in celestial reference coordinates

Fig. 2. Guidance by use of celestial reference coordinates when destination is beyond line-of-sight contact or direct radiation connection.

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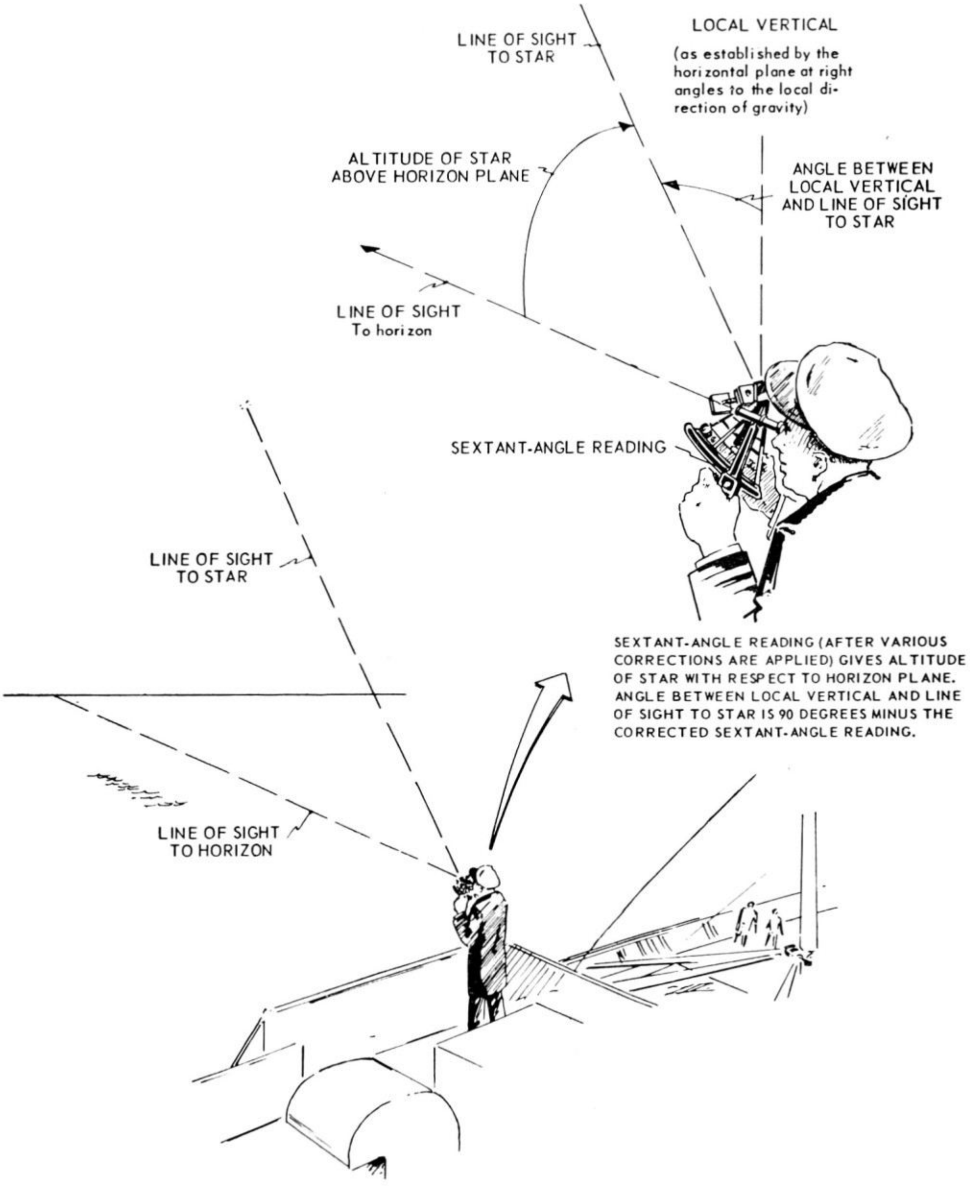
land masses and, at great distances, the Earth's curvature makes it impossible to see either the departure point or the destination. Radiation connections can be used far beyond the direct-line-of-sight ranges, but complicated and expensive equipment is required for guidance. In the past, this equipment did not exist and will certainly never be universally available. To meet the needs for guidance when line-of-sight and radiation connections are not possible, auxiliary co-ordinate systems may be used for guidance purposes. To be applied for this purpose, the auxiliary co-ordinate system must have two properties:

1. It must be accurately related in a known and usable way to some system of co-ordinates fixed to the Earth.
2. It must be suitable for practical determinations of present position in terms of its own co-ordinates.

When these two properties exist, present position is found with respect to the auxiliary system, which thus serves to provide reference co-ordinates. With the present position known in the reference space, the known relationship between the reference space and Earth co-ordinates is applied

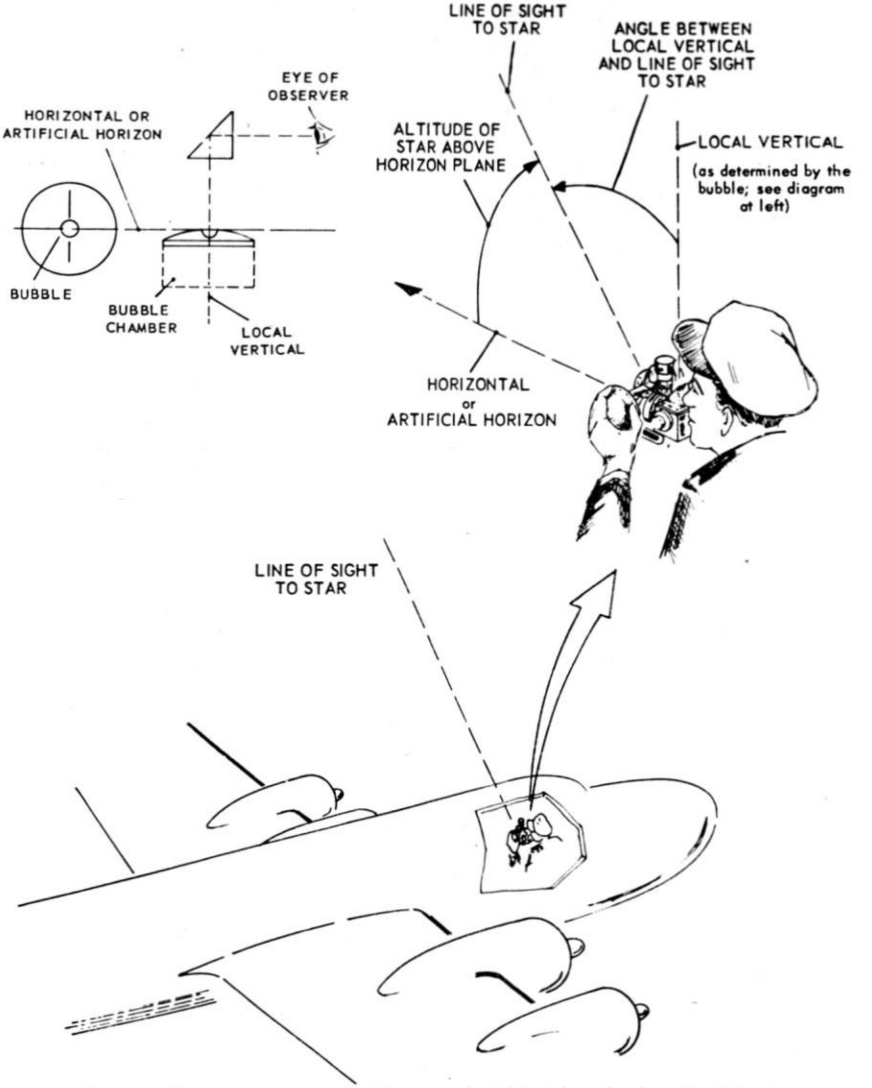


SEXTANT-ANGLE READING (AFTER VARIOUS CORRECTIONS ARE APPLIED) GIVES ALTITUDE OF STAR WITH RESPECT TO HORIZON PLANE. ANGLE BETWEEN LOCAL VERTICAL AND LINE OF SIGHT TO STAR IS 90 DEGREES MINUS THE CORRECTED SEXTANT-ANGLE READING.

c) Navigator's measurement of angles between celestial body lines of sight and local direction of gravity

Fig. 2. Guidance by use of celestial reference coordinates when destination is beyond line-of-sight contact or direct radiation connection.

to establish present position in these latter co-ordinates. Comparison of this present position with the programmed position on the Earth gives the correction components that are to be forced toward zero by the guidance system.



d) Use of bubble octant to establish angle between celestial-body line of sight and local direction of gravity

Fig. 2. Guidance by use of celestial reference coordinates when destination is beyond line-of-sight contact or direct radiation connection.

Celestial space as established by the "fixed stars" has been applied to navigational problems for a very long time. The stars are particularly suitable for guidance reference purposes because they are universally visible in clear weather and full information on their positions is well known and readily available as almanac data. Earth motion with respect to the celestial sphere is also well known and available. Figure 2a illustrates the determination of present position by the use of star lines of sight and celestial reference co-ordinates. Figure 2b shows that the essential observations determine the angles between the local direction of gravity at the present position and the stellar lines of sight. Figures 2c and 2d illustrate the way in which a sextant or an octant is used by a human observer to measure the line-of-sight data needed to fix present position with respect to the celestial sphere. The instantaneous position of the latitude and longitude co-ordinates of the Earth with respect to the celestial sphere is fixed by the instant or time at which observations are made. By use of almanac data on stars and a knowledge of time from chronometer readings, it is a routine matter to transfer present position from celestial co-ordinates to Earth co-ordinates.

INERTIAL REFERENCE CO-ORDINATES FOR GUIDANCE

Celestial space does not provide reference co-ordinates that are completely satisfactory for guidance purposes because they are available only under the special circumstances that allow reliable observations of stars. Overcasts, noctilucent clouds, daylight, too-close proximity to solar-system bodies, aerodynamic boundary layers on the guided vehicle, and imperfect windows combine to make celestial-body observations inaccurate or impossible. Special equipment may partially overcome these troubles by the addition of some complexity, but it is not likely that a completely satisfactory solution will be reached in equipment of reasonable size and cost.

The line-of-sight observation difficulties that are associated with the use of celestial space to provide auxiliary reference co-ordinates for guidance purposes may be eliminated by the use of instrumentally established inertial reference co-ordinates. This approach is feasible because Newton's law of dynamics shows that the acceleration of a mass particle in response to an applied force occurs with respect to inertial space, which is effectively identical with celestial space. Gyroscopic rotors make it possible to establish an artificial inertial reference space within vehicle-carried guidance equipment. This inertial reference space may be given any desired orientation in celestial co-ordinates and has the very great advantage of giving continuous operation for guidance purposes, whether or not lines of sight to celestial bodies are available. Figure 3 illustrates the basic features of a configuration in which an *inertial reference package* holds an initially established orientation with respect to celestial space. This orientation is shown with the supporting axis of the package set parallel to the Earth's axis of rotation.

With the arrangement of Fig. 3, two sets of auxiliary co-ordinates are effectively applied in solving the guidance problem. The reference co-ordinates that are directly used in determining present position are fixed

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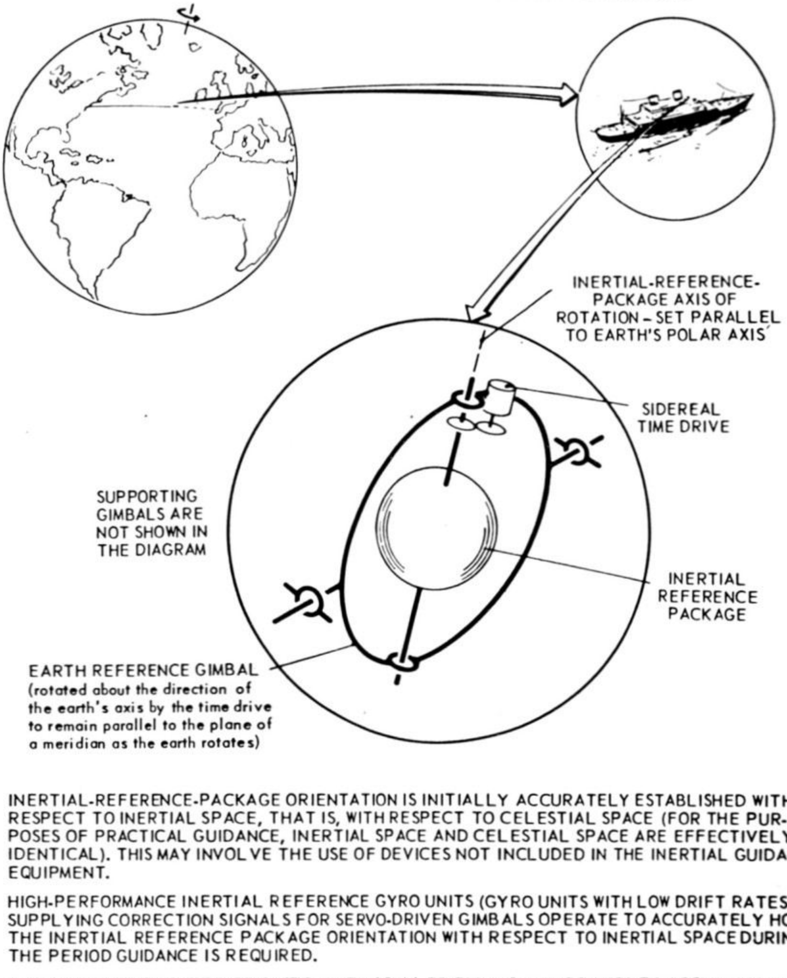

INERTIAL-REFERENCE-PACKAGE ORIENTATION IS INITIALLY ACCURATELY ESTABLISHED WITH RESPECT TO INERTIAL SPACE, THAT IS, WITH RESPECT TO CELESTIAL SPACE (FOR THE PURPOSES OF PRACTICAL GUIDANCE, INERTIAL SPACE AND CELESTIAL SPACE ARE EFFECTIVELY IDENTICAL). THIS MAY INVOLVE THE USE OF DEVICES NOT INCLUDED IN THE INERTIAL GUIDANCE EQUIPMENT.

HIGH-PERFORMANCE INERTIAL REFERENCE GYRO UNITS (GYRO UNITS WITH LOW DRIFT RATES) SUPPLYING CORRECTION SIGNALS FOR SERVO-DRIVEN GIMBALS OPERATE TO ACCURATELY HOLD THE INERTIAL REFERENCE PACKAGE ORIENTATION WITH RESPECT TO INERTIAL SPACE DURING THE PERIOD GUIDANCE IS REQUIRED.

A SPECIFIC FORCE RECEIVER SYSTEM WITH SCHULER TUNING MAY BE USED TO ACCURATELY INDICATE THE VERTICAL IN MOVING VEHICLES.

MEASUREMENT OF THE ANGLES BETWEEN THE VERTICAL AND A MEMBER FIXED WITH THE PROPER ORIENTATION TO THE EARTH REFERENCE GIMBAL GIVES PRESENT POSITION OF THE VEHICLE ON THE EARTH.

Fig. 3. Guidance by substitution of inertial reference coordinates for celestial coordinates.

to the mechanical component that carries the inertial reference package. These co-ordinates are given an initial orientation with respect to celestial co-ordinates and are held in this orientation by components inside the package. The inertial space established by this instrumental means provides the auxiliary reference co-ordinates for guidance purposes. The conversion from these inertial reference co-ordinates to Earth co-ordinates is especially simple when the support axis for the inertial reference package is set parallel to the Earth's axis. This is true because it is only necessary to supply an accurate sidereal time signal to the drive between the package and the gimbal, which then causes the gimbal to be rotated so that it moves with the Earth. The plane of the gimbal then remains parallel to the plane of some meridian fixed to the actual Earth and acts as the *Earth reference member* from which the direction of gravity can be measured.

The Earth reference gimbal effectively establishes an Earth co-ordinate system within the guidance equipment and makes it possible to determine present positions of the vehicle by comparing the local direction of gravity with the artificially established Earth co-ordinate system. Various configurations for realizing indications of present position by inertial-guidance equipment are possible but the basic principles involved in each case are those just described.

Satisfactory inertial-guidance equipment has special requirements in that it depends on the practical realization of:

1. Means for maintaining the orientation with respect to inertial space of the inertial reference package carried by a moving vehicle within tolerance limits and over time periods that are compatible with the requirements of the guidance problem to be solved.
2. Means for accurately indicating the local direction of gravity within equipment subjected to the erratic rotations, linear accelerations, and gravitational components that accompany the operation of a moving vehicle.

The first of these requirements may be met by a servodriven stabilization system and gyro units with good performance as far as the ability to hold orientation with respect to inertial space is concerned. Units with this low-drift-rate characteristic are of primary importance for inertial equipment. Reduced to the simplest terms, inertial guidance is a refined form of dead-reckoning navigation in which good accuracy of present-position indications depends on reference-package co-ordinates that do not rotate with respect to celestial space.

The second special inertial system requirement applies particularly to equipment for use in winged or floating vehicles that ordinarily operate with gravitational-field effects balanced by externally applied forces. The instrumental problem to be solved is that of accurately determining the direction of gravity in the presence of linear accelerations that vary erratically in direction and magnitude. This is difficult to accomplish in practice because of the physical fact described by Einstein's principle of equivalence, which states that, in general, it is impossible to distinguish body force components due to linear acceleration from body force components due to gravitational-field effects. In the face of this situation, satisfactory indications of the local vertical, which by definition is identical with the direction of gravity, may be achieved by designing the vertical-indicating sub-system to have the proper dynamic characteristics. The nature of these characteristics and their theoretical background were described by Schuler in a series of articles⁽¹⁾. His great contributions to the field of inertial guidance are universally honored by describing the essential dynamic characteristic of vertical-indicating sub-systems as *Schuler tuning*.

The nature of Schuler tuning and the reasons for its use are discussed in later sections of this paper.

The basic problems of low-drift-rate gyro units and other special components for inertial guidance systems are described in another presentation⁽²⁾ by the authors.

GENERAL FEATURES OF AN INERTIAL GUIDANCE SYSTEM

Inertial-system operation depends on the availability of four mechanism components:

1. An *inertial reference package*, which provides signals depending on the deviation of the package from an orientation that is non-rotating with respect to inertial space.
2. A *specific force receiving package*, which provides signals depending on the resultant of gravitational forces and inertia reaction forces due to the linear acceleration with respect to inertial space that acts on the package.
3. A *time signal generator*, which gives an output accurately representing sidereal time.
4. An *indicating system*, which (a) shows the change in direction of the local vertical with respect to a direction related to the point of departure by the inertial reference package and sidereal time, and (b) generates guidance information by comparing the angle between these two directions with a guidance program.

These four components may be combined in a number of different ways to form inertial guidance systems of various types. Wrigley, Woodbury and Hovorka⁽³⁾ have described several of the possible systems and have outlined their basic operating characteristics. The generalized discussion given by these authors will not be reviewed in detail here, as it is not necessary for the purpose of this paper which is to call attention to basic principles rather than to give a comprehensive description of details. This purpose is served by calling attention to typical problems of sub-system performance and component behavior in terms of an illustrative system.

MECHANICAL FEATURES OF AN ILLUSTRATIVE INERTIAL GUIDANCE SYSTEM

Figure 4 is a line schematic diagram showing the essential elements of the mechanical sub-system for an illustrative inertial guidance system in which the inertial reference package and the specific force receiving package are both rigidly connected to a controlled member. This controlled member is mounted on a base by means of a three-gimbal system that allows the controlled member complete angular freedom with respect to the base. The system includes three single-axis servomotor drives: one between the outer gimbal support and the outer gimbal*; one between the outer gimbal and the middle gimbal; and one between the middle gimbal and the inner gimbal, which carries the controlled member. The geometrical actions that occur when a configuration of this type operates are discussed in a paper by Draper and Woodbury⁽⁴⁾. In order to realize a working system, the output signals from the three gyro units of the inertial reference package, each of which is sensitive to rotation about a single axis only, must be properly distributed to three separate power control systems† associated with the servo motors. The three gyro units are mounted so that their input axes are at right angles to each other. The required distribution of signals to the power control systems and their associated motors is accomplished by the middle gimbal resolvers and inner gimbal resolver shown in Fig. 4. The theoretical relationships that must be fulfilled by the signals are given by Draper and Woodbury⁽⁴⁾. The resultant signals give the power control system for each of the servo motors and input command that causes all the drives to work together in forcing the controlled member toward alignment with a reference orientation established by the gyro units of the inertial reference package.

The *specific force receiving package* of Fig. 4 consists of two single-direction-sensitivity specific force receiver units with their input axes at right angles to each other in a plane perpendicular to the axis of the inner gimbal. Because the function of the specific force receiving package in the guidance system is to sense specific force‡ and supply signals that are used to cause a direction fixed in the controlled member to search out the direction of gravity, which defines the direction of the local vertical, this package is also called the *indicated vertical package*.

Figure 5 is a functional diagram for an inertial guidance system based on the mechanical arrangement of Fig. 4. Each of the three gyro units of the inertial reference package, by interactions among its internal components that will be described in a following section, generates a signal proportional to the inertial-space rotation of its case about the input axis

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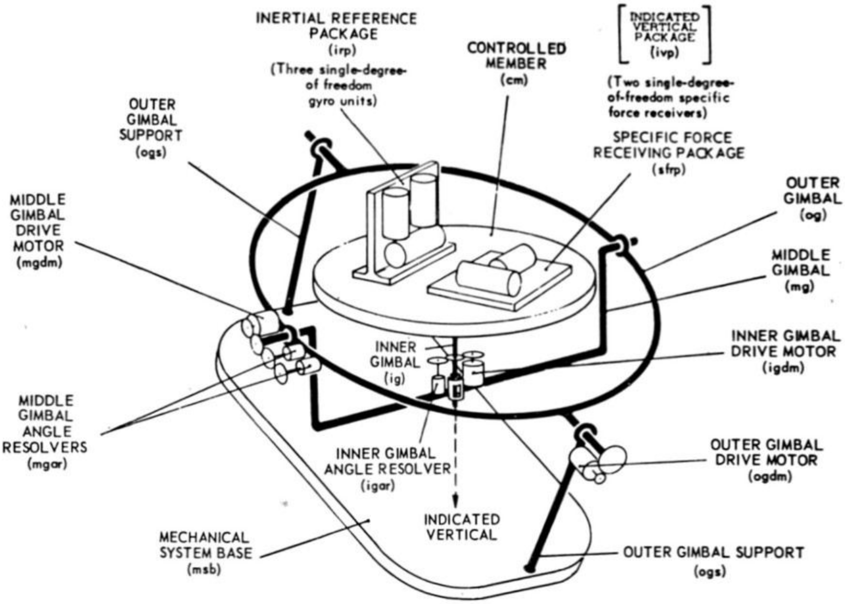
* In Fig. 4, the various gimbals are shown with rudimentary forms in order to illustrate the mechanical relationships involved. Practical systems employ closed-frame structures designed for maximum stiffness with minimum weight.

† These power control systems operate to supply the necessary electrical inputs for properly driving the servo motors. The power control system components are not represented in Fig. 4.

‡ By definition, specific force is the force per unit mass on a body due to the resultant of inertia reaction effects and gravitational effects. It is equal in magnitude and opposite in direction to the resultant acceleration acting on a body when gravity is represented by an equivalent acceleration.



NOTES: 1. THE BASE MOTION ISOLATION GIMBAL SYSTEM IS MADE UP OF THE OUTER GIMBAL SUPPORTS, THE OUTER GIMBAL, THE MIDDLE GIMBAL, THE INNER GIMBAL, THE ASSOCIATED DRIVE MOTORS AND THE ASSOCIATED RESOLVERS.

2. THE ELECTRICAL POWER SUPPLIES, ELECTRONIC UNITS, COMPUTERS, CONNECTIONS, RACKS AND OTHER COMPONENTS NECESSARY TO COMPLETE AN INERTIAL GUIDANCE SYSTEM ARE NOT REPRESENTED IN THIS FIGURE.

Fig. 4. Line schematic diagram showing the essential mechanical elements of an inertial guidance system based on rotation of the inertial reference package with the indicated vertical.

with respect to a *reference orientation* of the case. This reference orientation is non-rotating with respect to inertial space unless it is changed by an input command signal to the gyro unit. Because of the actions of the individual gyros, the inertial reference package, with its three units, produces signals that depend on the orientational deviation of the package from a reference orientation. These signals represent the correction to the orientation of the inertial reference package that must be made in order to move the package into coincidence with its reference orientation. For this reason, the three signals may be considered as making up the *correction signal** for the controlled member to which the gyro units are attached. The controlled member correction signal is supplied to the *controlled member drive power control system*, and causes this system to give the proper power to each of the gimbal servo-drive motors for moving the controlled member so that the inertial reference package orientation hunts close to coincidence with its reference orientation. Except for causing the inertial reference package to follow reference-orientation changes, the action of the gyro

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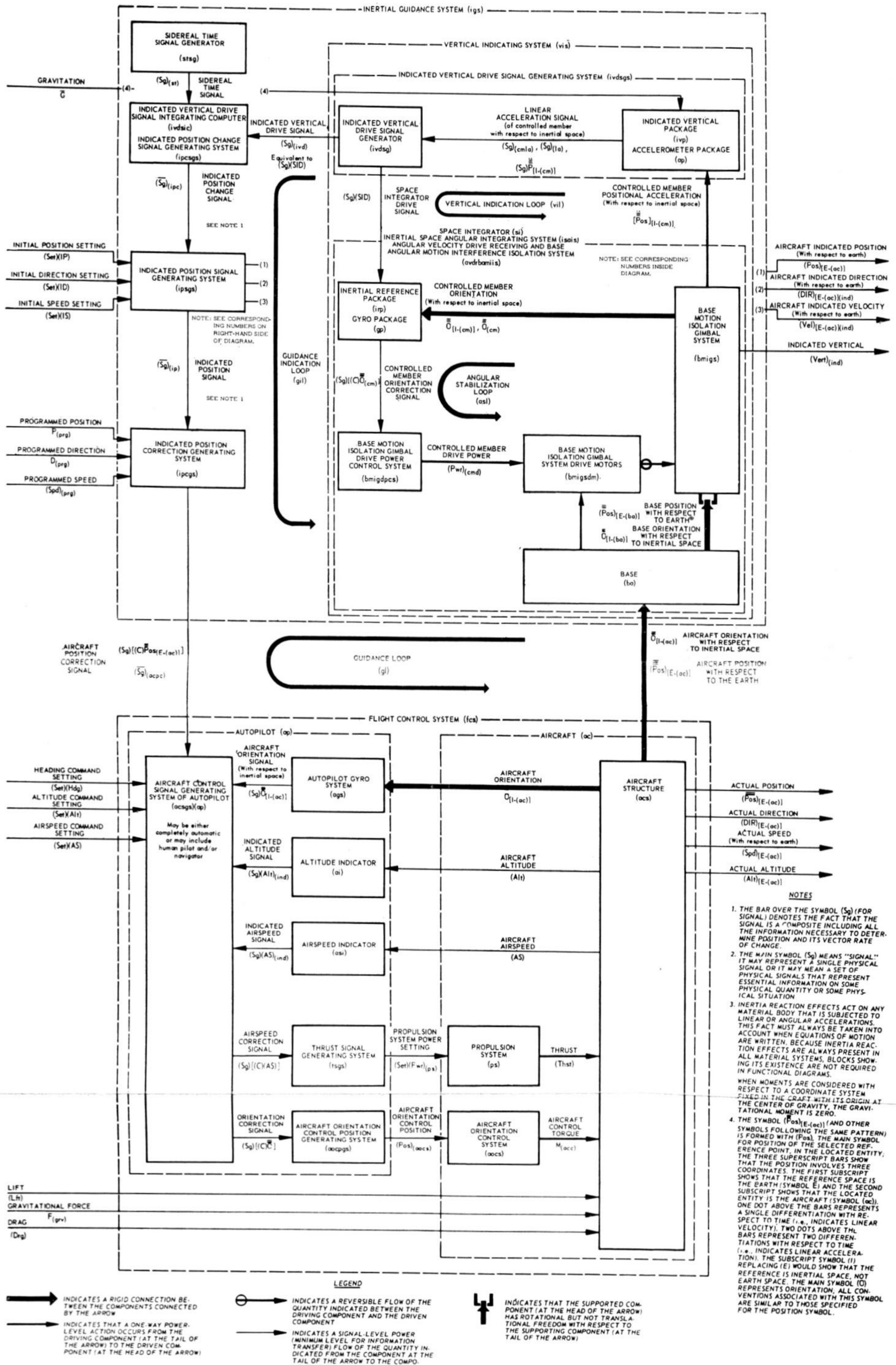
* For convenience in generalized discussions, a set of any number of individual signals that represent information on a single physical quantity may be treated as a composite and referred to as a single signal.



NOTES

1. THE BAR OVER THE SYMBOL (Sg) (FOR SIGNAL) DENOTES THE FACT THAT THE SIGNAL IS A COMPOSITE INCLUDING ALL THE INFORMATION NECESSARY TO DETERMINE POSITION AND ITS VECTOR RATE OF CHANGE.
2. THE MAIN SYMBOL (Sg) MEANS "SIGNAL." IT MAY REPRESENT A SINGLE PHYSICAL SIGNAL OR IT MAY MEAN A SET OF PHYSICAL SIGNALS THAT REPRESENT ESSENTIAL INFORMATION ON SOME PHYSICAL QUANTITY OR SOME PHYSICAL SITUATION
3. INERTIA REACTION EFFECTS ACT ON ANY MATERIAL BODY THAT IS SUBJECTED TO LINEAR OR ANGULAR ACCELERATIONS. THIS FACT MUST ALWAYS BE TAKEN INTO ACCOUNT WHEN EQUATIONS OF MOTION ARE WRITTEN. BECAUSE INERTIA REACTION EFFECTS ARE ALWAYS PRESENT IN ALL MATERIAL SYSTEMS, BLOCKS SHOWING ITS EXISTENCE ARE NOT REQUIRED IN FUNCTIONAL DIAGRAMS.

   WHEN MOMENTS ARE CONSIDERED WITH RESPECT TO A COORDINATE SYSTEM FIXED IN THE CRAFT WITH ITS ORIGIN AT THE CENTER OF GRAVITY, THE GRAVITATIONAL MOMENT IS ZERO.
4. THE SYMBOL $(\overline{\overline{\overline{P}}}os)_{[E-(ac)]}$ (AND OTHER SYMBOLS FOLLOWING THE SAME PATTERN) IS FORMED WITH (Pos), THE MAIN SYMBOL FOR POSITION OF THE SELECTED REFERENCE POINT, IN THE LOCATED ENTITY; THE THREE SUPERSCRIPT BARS SHOW THAT THE POSITION INVOLVES THREE COORDINATES. THE FIRST SUBSCRIPT SHOWS THAT THE REFERENCE SPACE IS THE EARTH (SYMBOL E) AND THE SECOND SUBSCRIPT SHOWS THAT THE LOCATED ENTITY IS THE AIRCRAFT (SYMBOL (ac)). ONE DOT ABOVE THE BARS REPRESENTS A SINGLE DIFFERENTIATION WITH RESPECT TO TIME (i.e., INDICATES LINEAR VELOCITY). TWO DOTS ABOVE THE BARS REPRESENT TWO DIFFERENTIATIONS WITH RESPECT TO TIME (i.e., INDICATES LINEAR ACCELERATION). THE SUBSCRIPT SYMBOL (I) REPLACING (E) WOULD SHOW THAT THE REFERENCE IS INERTIAL SPACE, NOT EARTH SPACE. THE MAIN SYMBOL (O) REPRESENTS ORIENTATION; ALL CONVENTIONS ASSOCIATED WITH THIS SYMBOL ARE SIMILAR TO THOSE SPECIFIED FOR THE POSITION SYMBOL.

LEGEND

INDICATES A RIGID CONNECTION BETWEEN THE COMPONENTS CONNECTED BY THE ARROW

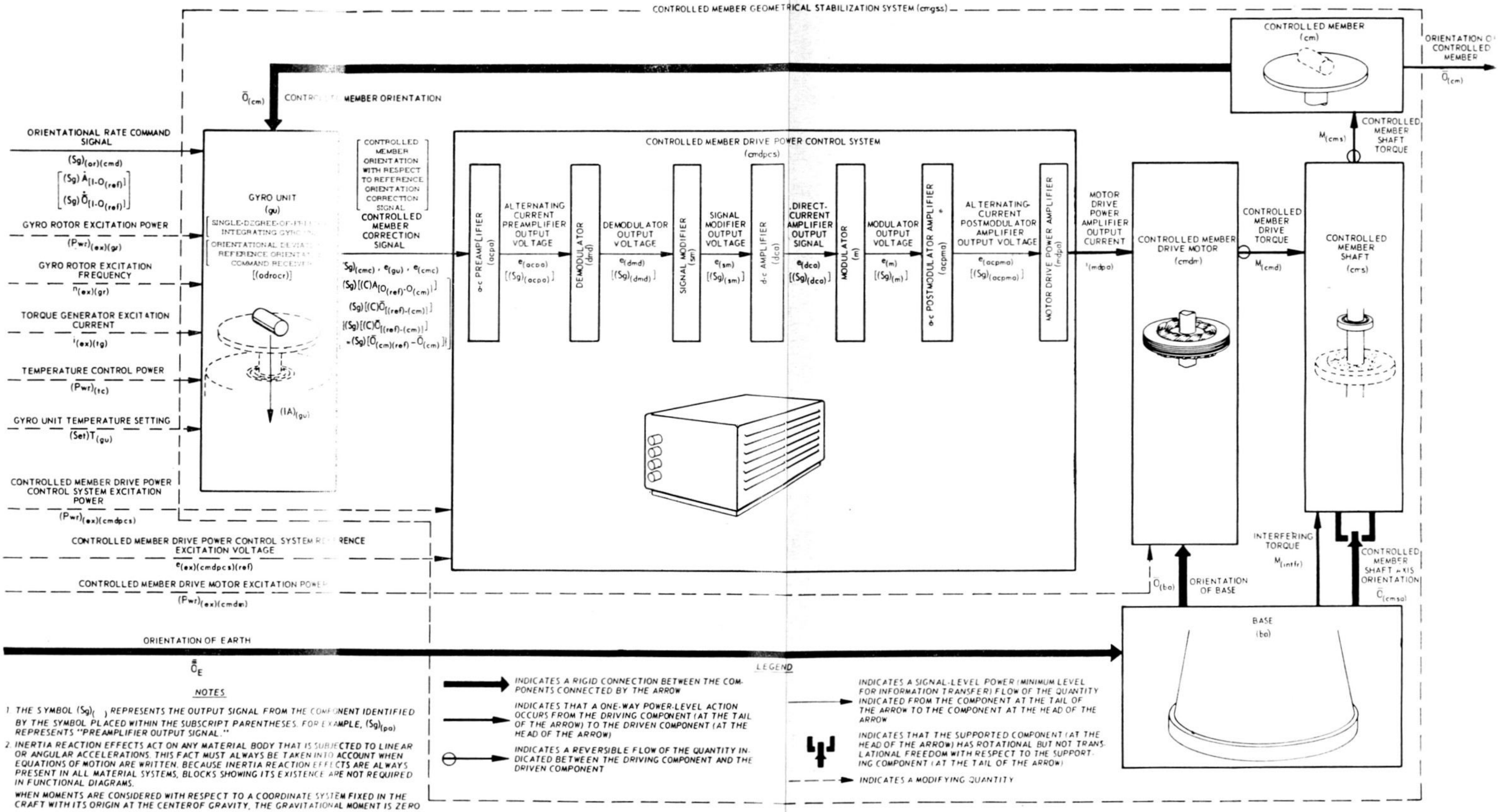
INDICATES THAT A ONE-WAY POWER-LEVEL ACTION OCCURS FROM THE DRIVING COMPONENT (AT THE TAIL OF THE ARROW) TO THE DRIVEN COMPONENT (AT THE HEAD OF THE ARROW)

INDICATES A REVERSIBLE FLOW OF THE QUANTITY INDICATED BETWEEN THE DRIVING COMPONENT AND THE DRIVEN COMPONENT

INDICATES A SIGNAL-LEVEL POWER (MINIMUM LEVEL FOR INFORMATION TRANSFER) FLOW OF THE QUANTITY INDICATED FROM THE COMPONENT AT THE TAIL OF THE ARROW TO THE COMPONENT AT THE HEAD OF THE ARROW.

INDICATES THAT THE SUPPORTED COMPONENT (AT THE HEAD OF THE ARROW) HAS ROTATIONAL BUT NOT TRANSLATIONAL FREEDOM WITH RESPECT TO THE SUPPORTING COMPONENT (AT THE TAIL OF THE ARROW)

Fig. 5. Functional diagram showing essential subsystems and components of an illustrative inertial guidance system based on continuous alignment of the inertial package with the indicated vertical.



NOTES

1. *THE SYMBOL $(Sg)_{(\ )}$ REPRESENTS THE OUTPUT SIGNAL FROM THE COMPONENT IDENTIFIED BY THE SYMBOL PLACED WITHIN THE SUBSCRIPT PARENTHESES. FOR EXAMPLE, $(Sg)_{(pa)}$ REPRESENTS "PREAMPLIFIER OUTPUT SIGNAL."*
2. *INERTIA REACTION EFFECTS ACT ON ANY MATERIAL BODY THAT IS SUBJECTED TO LINEAR OR ANGULAR ACCELERATIONS. THIS FACT MUST ALWAYS BE TAKEN INTO ACCOUNT WHEN EQUATIONS OF MOTION ARE WRITTEN. BECAUSE INERTIA REACTION EFFECTS ARE ALWAYS PRESENT IN ALL MATERIAL SYSTEMS, BLOCKS SHOWING ITS EXISTENCE ARE NOT REQUIRED IN FUNCTIONAL DIAGRAMS.*

WHEN MOMENTS ARE CONSIDERED WITH RESPECT TO A COORDINATE SYSTEM FIXED IN THE CRAFT WITH ITS ORIGIN AT THE CENTEROF GRAVITY, THE GRAVITATIONAL MOMENT IS ZERO

LEGEND

➡	*INDICATES A RIGID CONNECTION BETWEEN THE COMPONENTS CONNECTED BY THE ARROW*
→	*INDICATES THAT A ONE-WAY POWER-LEVEL ACTION OCCURS FROM THE DRIVING COMPONENT (AT THE TAIL OF THE ARROW) TO THE DRIVEN COMPONENT (AT THE HEAD OF THE ARROW)*
⊖→	*INDICATES A REVERSIBLE FLOW OF THE QUANTITY INDICATED BETWEEN THE DRIVING COMPONENT AND THE DRIVEN COMPONENT*
→	*INDICATES A SIGNAL-LEVEL POWER (MINIMUM LEVEL FOR INFORMATION TRANSFER) FLOW OF THE QUANTITY INDICATED FROM THE COMPONENT AT THE TAIL OF THE ARROW TO THE COMPONENT AT THE HEAD OF THE ARROW*
┗┳┛	*INDICATES THAT THE SUPPORTED COMPONENT (AT THE HEAD OF THE ARROW) HAS ROTATIONAL BUT NOT TRANSLATIONAL FREEDOM WITH RESPECT TO THE SUPPORTING COMPONENT (AT THE TAIL OF THE ARROW)*
- - →	*INDICATES A MODIFYING QUANTITY*

THE DIAGRAM OF THIS FIGURE ILLUSTRATES THE FUNCTIONAL RELATIONSHIPS AMONG THE OPERATING COMPONENTS FORMING A SINGLE-AXIS SPACE ANGULAR INTEGRATING SYSTEM.

Fig. 14. Functional diagram for an illustrative single-axis controlled member inertial-space geometrical stabilization system.

units, the power control system and the servo-driven gimbal system is to hold the controlled member nonrotating, or *stabilized*, with respect to inertial space, no matter how the base of the system may rotate. For this reason, the closed chain formed by the operating components mentioned is called the *angular stabilization loop*.

An essential characteristic of the gyro units that make up the inertial reference package is that they are capable of receiving electrical-signal command inputs that cause their reference positions to have a controlled rotation with respect to inertial space about their input axes. The system is designed so that the rate of rotation of the reference position with respect to inertial space is proportional to the magnitude of the command signal. As a result of this action, the total angle through which the reference position rotates during any given time interval is proportional to the time integral of the command signal acting during the interval. The effect of the three command signals applied to the three gyro units of the inertial reference package is to cause a resultant rotation of the reference orientation of the package whose direction and magnitude represent the time integral of the resultant of the command signals. The angular stabilization loop of Fig. 5 operates to keep the change in orientation of the controlled member effectively identical with the change in the reference orientation of the inertial reference package. This means that the components that make up the stabilization loop act as a three-axis *inertial space angular integrating system*, which may also be called a *space integrator* when a misunderstanding is not likely.

The two units of the specific force receiving package shown in Fig. 4 are rigidly attached to the controlled member so that they generate signals proportional to the specific forces acting along two directions at right angles to the inner gimbal axis. These two signals form a composite specific force signal representing the resultant of linear acceleration forces in a plane normal to the axis of the inner gimbal, and the projection of gravitational force on this plane. This specific force signal is the input for the *indicated vertical drive signal generator*, which supplies the input signal for the space integrator. The response of the space integrator to this signal is to rotate a direction fixed to the controlled member into substantial coincidence with the *local direction of gravity*, that is, the direction in which an accurate plumb bob would hang on a stationary base at the instantaneous location of the guidance system. The *indicated vertical* is the direction of the inner gimbal axis and is along the local true vertical for perfect operation of the guidance system, whether the system is at rest or is moving. With the system at rest on the Earth, the space integrator drive signal serves its function as it causes the gimbal drive motors to move the controlled member into the orientation in which both specific force indicators give zero output signals.

Indications of the vertical by a servo, gimbal and platform combination can also be achieved if single-axis pendulum units are used to provide the specific force receiver functions. With this arrangement, each pendulum unit includes a signal generator to produce signals when the arm of the *pendulous mass* is not along an *arm reference axis* fixed to the case of the unit. The *input axis* for the pendulum is a direction at right angles to the arm reference axis and to the axis about which the pendulum is pivoted. With this configuration, a single-axis pendulum acts as a receiver for specific force along its input axis, but disregards specific forces along all other directions as long as the arm of its unbalance mass is substantially along the arm reference axis. This means that under conditions existing for the specific force receivers of an indicated vertical package either single-axis pendulums or single-axis specific force receiver units will serve the required purpose. With the guidance system fixed to the Earth, the pendulums will hang down along the direction of gravity (which is the *local vertical* direction) and will give zero output signals only when their arm reference axes (fixed on the controlled member parallel to the indicated vertical direction) are along the vertical. When the arms are not along the true vertical, the pendulums will produce output signals that will cause the space integrator to rotate the indicated vertical direction into coincidence with the true vertical.

When the base of the guidance system is carried in a moving vehicle, the pendulums* are subjected to specific forces that pull their pendulous-mass arms away from the arm reference axes and cause the output signals to change from zero. These deflecting specific force components come from two sources:

1. Gravity, when the controlled member is tipped so that the indicated vertical requires a correction angle to move it into coincidence with the true vertical.
2. Linear accelerations acting in the horizontal plane.

The gravitational-force components act to correct the indicated vertical, but the horizontal-plane linear-force components, which in modern aircraft are of considerable magnitude in comparison with gravity and continue for relatively long time intervals, act to cause intolerably great errors of the indicated vertical unless the equipment involved is properly designed. *The basic problem that must be solved by a satisfactory vertical indicating system is that of compensating for the interfering effects of linear-acceleration disturbances, so that the indicated vertical accurately follows the local direction of gravity when the system is carried by a moving vehicle as well as when it is stationary on the Earth.* The solution provided by Schuler tuning is discussed in the next section.

In order to realize satisfactory angular stabilization, the dynamics of the space integrator of Fig. 5 must be very much faster than the dynamics

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* Pendulums are used for the purposes of this discussion because they are adapted to give somewhat simpler pictures of the essential physical actions involved. It is to be understood that the discussion of pendulum operation applies equally well to systems using single-axis linear specific force receiver units.

associated with Schuler tuning. This means that the *indicated vertical drive signal generating system* will determine the over-all dynamics of the vertical indicating system, and may be designed as necessary for the realization of Schuler tuning, without regard for the necessary function of stabilization to eliminate interference due to base rotation.

Starting from the specific force receiving package signal and ending with a change in the direction of the indicated vertical that corresponds to a displacement of the guidance system over the Earth, the effect of the various actions that occur in the vertical indicating system is equivalent to a double integration of the accelerometer-package signal. One of these integrations is accurately carried out by the space integrator, leaving one integration to be accomplished by the indicated vertical drive signal generating system. Many arrangements are available for realizing this necessary single integration and adjusting the system dynamics to minimize dynamic errors of the indicated vertical. A number of these ways are discussed by Wrigley, Woodbury and Hovorka(3), who include a list of pertinent references. From the standpoint of mechanization, the components required to realize any possible performance for the indicated vertical drive signal generating system are similar to the components ordinarily used in electronic and servomechanism equipment. The characteristics associated with components of this kind are commonly known and do not present any extraordinary problems so far as the present discussion is concerned.

As shown by Fig. 5, the *vertical indicating system*, which is formed by the *indicated vertical drive signal generating system* and the *space integrator*, has the position and orientation of the aircraft as its inputs and produces the *indicated vertical and the indicated vertical drive signal* as its outputs. The essential element for realizing inertial guidance in practice is to provide an integrating computer as part of the equipment. This computer should have the same integration performance as the space integrator containing the inertial reference package. By supplying the integrating computer with the indicated vertical drive signal as its input, an output representing the resultant angular change in the direction of the indicated vertical during any elapsed time period will be generated. The signal representing this angular displacement will be a measure of the distance moved by a craft carrying the equipment over a non-rotating Earth. With the equipment stationary on the actual Earth, which is rotating at the rate of one revolution in 24 sidereal hours, the integrating computer will give an output corresponding to an indication of sidereal time. Because the angular velocity of the Earth is accurately known, it is possible to subtract the effect of the Earth's rotation from the integral of the indicated vertical drive signal on the basis of an input to the integrating computer from a sidereal time signal generator.

The science of timing devices based on tuning forks or crystal oscillators is well developed and equipment of reasonable size capable of meeting the requirements of inertial guidance is generally available (5–10). The problems associated with such equipment are well known and will not be discussed here.

The indicated vertical drive signal integrating computer of Fig. 5 produces a signal whose change during any given time interval represents the change in indicated position with respect to the Earth during the same interval. This indicated position change signal is the primary input for the *indicated position signal generating system*, which also accepts settings of *initial position*, *initial direction* and *initial speed*. This system combines the information on indicated position change with information on starting conditions to produce output signals that continuously represent the position, direction and speed of the aircraft carrying the guidance system.

The *indicated position signal** is one of the inputs to the *indicated position correction generating system*, which also receives programmed position, programmed direction and programmed speed from a source outside of the system represented by the functional diagram of Fig. 5. This system compares instantaneous states of the programmed quantities with the actual states of the same quantities and produces a composite signal representing the corrections to the actual quantities that are required to bring them into agreement with their programmed states.

The techniques and components required to realize indicated position signal generating systems and indicated position correction generating systems are generally similar to those commonly used in computers and do not require further attention here.

Figure 5 shows that the output of the *inertial guidance system*, which functionally combines the stabilization loop and the vertical indication loop to form the *guidance indicating loop*, is the *aircraft position correction signal*. This signal contains the information necessary to change the direction and speed of the aircraft so as to cause it to approach the programmed state of these quantities. The aircraft position correction signal is the primary input for the autopilot that produces the inputs necessary to operate the orientation control system of the aircraft and to adjust the power setting of the propulsion system. The *flight control system*, whose basic function is to determine the path and speed of the vehicle carrying the inertial guidance system, is made up of the autopilot and the aircraft itself. The functional relationships involved in the operation of flight control systems are discussed by Draper(11) who also gives a list of references on this subject. The outputs of the flight control system are the position of the aircraft with respect to the Earth and the orientation of the aircraft with respect to inertial space. These quantities are transferred to the base of the mechanical system of the guidance system by a rigid mounting of this base to the structure of the aircraft. This mechanical

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* The indicated position signal is a composite signal representing not only aircraft position but also the vector rate of change of this position.

connection closes the *guidance loop* formed by the combination of the aircraft and the inertial guidance system. The essential features of aircraft and automatic pilots are common knowledge and do not require special treatment from the standpoint of instrumentation problems. However, it is important to note that the flight control system must be carefully engineered and built in order to provide response characteristics of the high quality necessary if acceptable performance is to be realized from an inertial guidance system.

GEONAVIGATIONAL FACTORS—SHAPE OF THE EARTH(12, 13, 14)

According to Newton's law of gravitation, a massive body such as the Earth can be considered as having associated with it a gravitational field. The spatial direction of this field at any given point on the Earth might serve as a unique identification of the position of that point. However, it is impossible to distinguish directly between gravitational forces and inertial forces, such as the centrifugal force due to the Earth's daily rotation. This is expressed in a basic law of physics, namely the principle of equivalence in the general theory of relativity, that gravitational mass and inertial mass are equivalent(15)*. The vector resultant of the Earth's

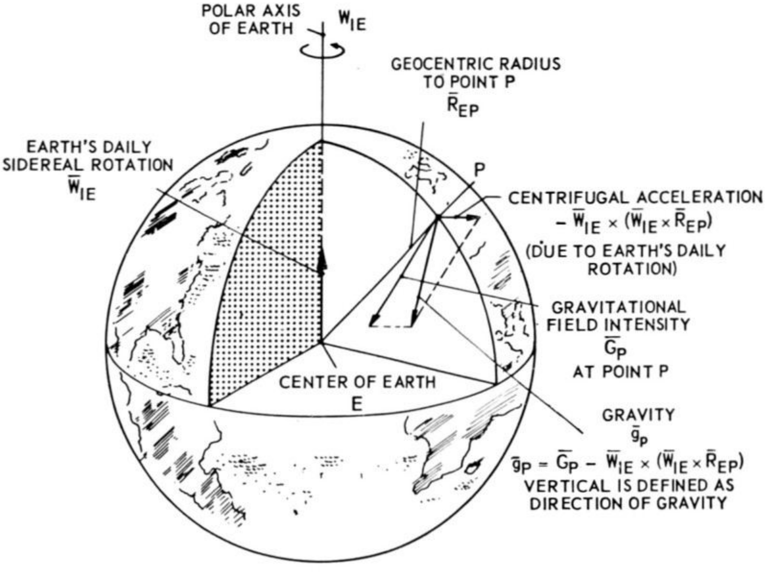

Fig. 6. Relationship of factors to produce the vertical.

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* Here, *gravitational mass* is the mass concept inherent in the inverse-square law of mass attraction, while *inertial mass* is the mass concept inherent in Newton's second law, that the net force is given by the product of mass and acceleration.

gravitational field and the centrifugal force per unit mass due to the Earth's daily rotation is defined as the Earth's gravity field (see Fig. 6). The spatial direction of this gravity field is nearly radial and, neglecting anomalies, is unique at any given point on the Earth. The secular (time) variation in the direction of the gravity field at a given point, caused mainly by tidal effects, is less than 0·05 microradian(12). Thus, for navigational purposes, the direction of the gravity field is a reliable, unique characteristic of any given point on the Earth. Furthermore, it is essentially impossible to interfere with gravity effects.

The Earth itself is also subject to the equivalence of gravitational and inertial effects. Because of daily rotation, the associated centrifugal force field causes the Earth to bulge at the equator, producing an ellipsoidal shape. The figure that a fluid body with the mass distribution and daily rotation of the Earth would have is defined as *the geoid.* The surface of the geoid is represented by mean sea level, and variations in the elevation of the geoid relative to the closest reference ellipsoid are approximately 1% of the topographic variations in elevation. The reference ellipsoid, having an ellipticity* of 1/297, is seen to depart only slightly from a sphere. *The direction of the force of gravity, which is the gradient of the gravity potential at the surface of the geoid, is defined as the vertical*; it is illustrated in Fig. 7.

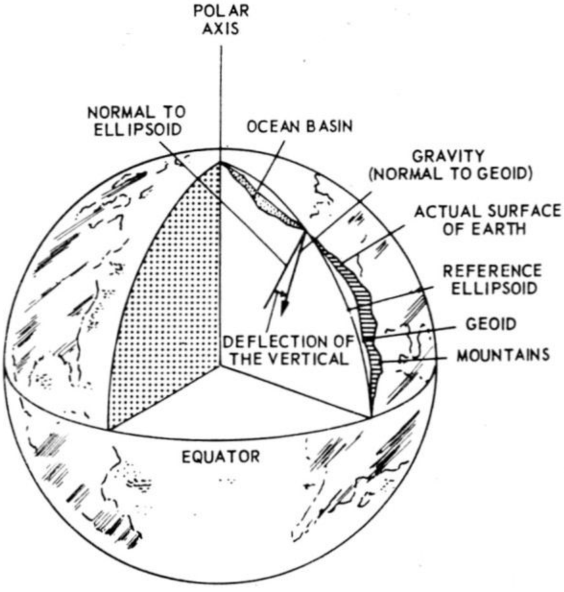

Fig. 7. Relationship of the geoid to a reference ellipsoid.

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* Ellipticity is the ratio of the difference between the equatorial radius and the polar semidiameter to the equatorial radius.

It is indicated most simply by a plumb bob whose base is stationary with respect to the Earth.

The geoid is not analytically smooth, because of local variations in the densities of the materials that make up the Earth's crust. This is further accentuated by the topographic variations in the Earth's surface. Such deviations are known as *gravity anomalies*. Because the geoid does not have an analytically smooth surface, the vertical is not in general parallel to the normal to the reference ellipsoid at the same position. This angular deviation, called *the deflection of the vertical* or *station error*, is generally less than 0·30 milliradian, and over a continental land mass rarely exceeds 0·10 milliradian. Figure 7 shows the relationship of the geoid to a reference ellipsoid.

INDICATION OF THE VERTICAL

The direction of the force of gravity, i.e. the vertical, is the basic physical factor that identifies the local position of any point on the Earth for inertial guidance systems. The measurement of this direction, the indication of the vertical, is an essential requirement on any such system. The vertical is very easily indicated by a simple plumb bob when the measurement is made from a base that is stationary on the Earth; but on a moving vehicle the roll, pitch and yaw of the craft, as well as its linear acceleration, would cause the plumb bob to depart markedly from the vertical.

Geocentric Rotation

The early "artificial horizon" technique for indicating the vertical implied a "flat Earth" model, for measurement purposes. That is, gravity was assumed to be constant in direction, while accelerations were considered to be either varying in direction or limited in time duration. Thus an indicator of the vertical became a mechanical low-pass filter. The difficulty in constructing a mechanical filter with a time parameter of several minutes, plus the fact that the flat-earth assumption is not correct, led to devices of an accuracy of about one-half degree or more during maneuvers. Isolation from roll, pitch and yaw influences on measurement was considerably better than isolation from vehicle maneuver effects, due to the fact that the fundamental periods of roll, pitch and yaw are much shorter than those of the maneuvers of a craft.

Any more precise treatment of the indication of the vertical from moving bases should use the concept of a geocentrically* rotating vertical. This was first recognized by Schuler(1) in his classic paper; the paper by Wrigley(16) expresses Schuler's thoughts in English. The geocentric-rotation concept expresses the acceleration of a point near the surface of

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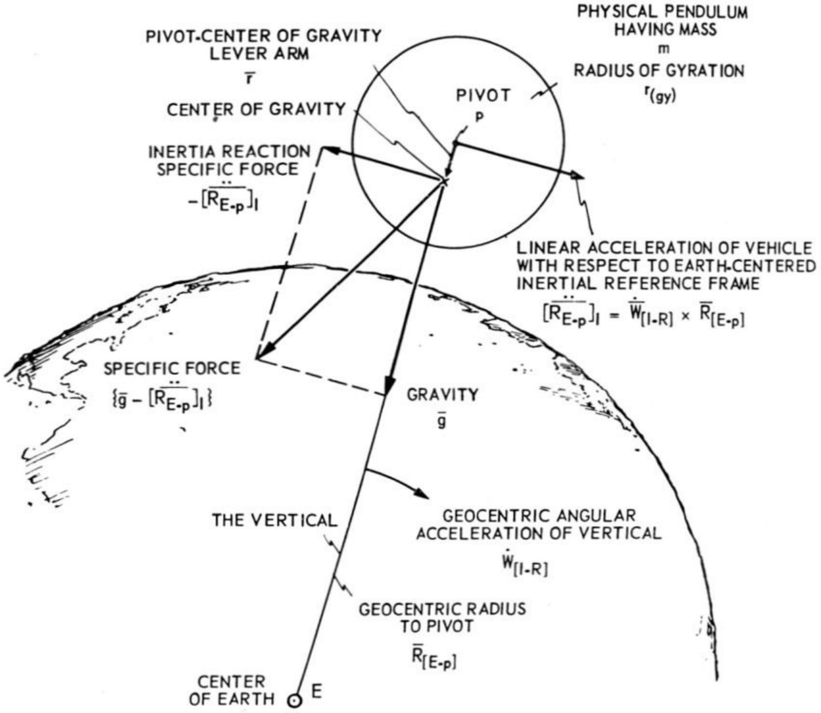
* Strictly speaking, this rotation is not truly geocentric, since the Earth is not actually spherical. However, this assumption does not restrict the validity of this discussion.

the Earth in terms of geocentric angular motions. In Derivation Summary 1, it is seen that only three of these geocentric angular accelerations are generally important:

(a) The tangential (horizontal) acceleration given a constant-altitude craft by a net force applied by its propulsion system. This acceleration is seen to be associated with an essentially geocentric angular acceleration of the vertical.



IDEAL SITUATION:

ARM $\bar{r}$ REMAINS PARALLEL TO GRAVITY $\bar{g}$

NONROTATING SPHERICAL EARTH ASSUMED FOR SIMPLICITY (DOES NOT INVALIDATE RESULTS)

APPLIED TORQUE DUE TO ACCELERATION $= m\bar{r} \times \{-[\ddot{\overline{R}}_{E\text{-}p}]_I\} = mr\,R_{[E\text{-}p]}\dot{\overline{W}}_{[I\text{-}R]}$

INERTIA REACTION OF PENDULUM $= mr^2_{(gy)}\dot{\overline{W}}_{[I\text{-}p]}$

$$mr^2_{(gy)}\dot{\overline{W}}_{[I\text{-}p]} = mr\,R_{[E\text{-}p]}\dot{\overline{W}}_{[I\text{-}R]}$$

FOR SCHULER TUNING, THE ANGULAR ACCELERATION OF THE PENDULUM MUST EQUAL THE GEOCENTRIC ANGULAR ACCELERATION OF THE VERTICAL; I.E.,

$$\dot{\overline{W}}_{[I\text{-}p]} = \dot{\overline{W}}_{[I\text{-}R]}$$

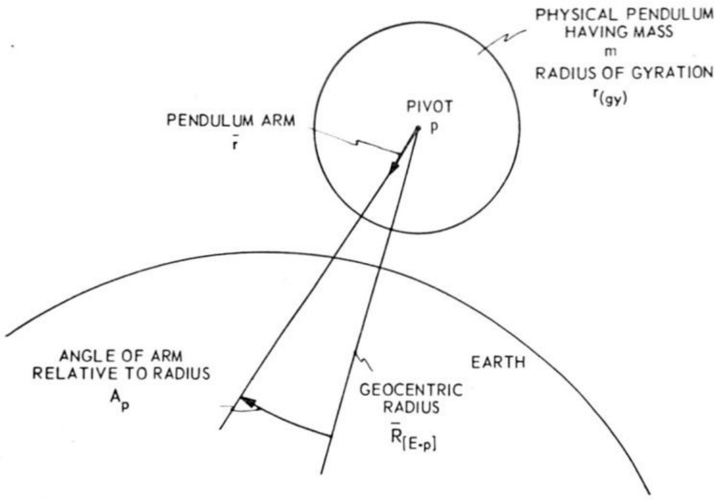
SO THAT $\boxed{\dfrac{r}{r^2_{(gy)}} = \dfrac{1}{R_{[E\text{-}p]}}}$ IS THE SCHULER TUNING CONDITION.

a) Schuler tuning condition in terms of physical-pendulum dimensions

Fig. 8. Simplified derivation of the Schuler tuning condition on a physical pendulum.

(b) The centrifugal acceleration, parallel to the equatorial plane, due to the Earth's daily rotation; this vector is combined with gravitation to produce gravity (see Fig. 6).

(c) Coriolis acceleration due to coupling of the craft's ground speed with the Earth's daily rotation; this must generally be compensated for.



GENERAL SITUATION:

ARM $\bar{r}$ NOT PARALLEL TO GRAVITY $\bar{g}$

APPLIED TORQUE $= m\bar{r} \times \{\bar{g} - [\overline{\ddot{R}_{E\text{-}p}}]_I\}$

$= m r R_{[E\text{-}p]} \dot{\bar{W}}_{[I\text{-}R]} - m r g \bar{A}_p$

INERTIA REACTION OF PENDULUM $= m r^2_{(gy)} \dot{\bar{W}}_{[I\text{-}p]}$

$= m r^2_{(gy)} \{\dot{\bar{W}}_{[I\text{-}R]} + \ddot{\bar{A}}_p\}$

$$\ddot{\bar{A}}_p + \frac{g r}{r^2_{(gy)}} \bar{A}_p = \left[\frac{r R_{[E\text{-}p]}}{r^2_{(gy)}} - 1 \right] \dot{\bar{W}}_{[I\text{-}R]}$$

BY SCHULER TUNING (THAT IS, $\frac{r}{r^2_{(gy)}} = \frac{1}{R_{[E\text{-}p]}}$),

$$\ddot{\bar{A}}_p + \frac{g}{R_{[E\text{-}p]}} \bar{A}_p = 0$$

SCHULER PERIOD

$$T_{ns} = 2\pi \sqrt{\frac{R_{[E\text{-}p]}}{g}} = 84.6 \text{ MINUTES}$$

b) Period of a schuler-tuned pendulum

Fig. 8. Simplified derivation of the Schuler tuning condition on a physical pendulum.

Fundamentals of the Schuler (84 min) *Pendulum*

When a vehicle moves over the nearly spherical Earth, the vertical associated with its instantaneous position rotates approximately geocentrically with respect to the Earth at an angular rate numerically equal in minutes of arc per hour to the vehicle ground speed in knots. If the Earth were a sphere, then, a pendulum on a vehicle moving at constant speed over the Earth would have to rotate with respect to the Earth at a constant angular velocity in order to indicate the vertical continuously. Once the transient stages were over, no torque would be required to keep the pendulum along the vertical during this rotation. On the other hand, if the vehicle were to accelerate, by changing either speed or heading, a torque would be required to maintain the pendulum on the vertical, which in turn is geocentrically accelerating.

Schuler(1) pointed out in 1923 that on a spherical Earth accurate indication of the vertical during periods of acceleration could be realized by suitable tuning of the natural period of the pendulum. Whenever the pivot of a pendulum is accelerated, the center of mass of the pendulum tends to "lag behind" the pivot with respect to inertial space. At the same time, this pivot acceleration causes the true vertical, which is a line through the center of the Earth and the pivot under the simplifying assumptions used in this treatment, to accelerate geocentrically with respect to inertial space. The rotational sense of these two motions is seen from Fig. 8 to be the same. These considerations lead to the following observation:

> If a pendulum initially hangs vertically, it will remain along the vertical if, as seen from inertial space, *its angular acceleration about its pivot equals the geocentric angular acceleration of the vertical.*

Figure 8 shows that these two angular accelerations become equal for a distributed-mass pendulum when the ratio of the displacement of the center of mass from the pivot to the square of the radius of gyration of the pendulum equals the reciprocal of the radius of the Earth. When operation is in the Earth's gravity field, and the pendulum is undamped, this condition gives a pendulum with a natural period of approximately 84 min; hence the term 84 min *pendulum*. For a shorter-period pendulum the rotation-producing torque on the pendulous element about its pivot is too strong, so the pendulum rotates too rapidly and lags the true vertical. For a longer-period pendulum the torque is too weak and the pendulum leads the true vertical. Unfortunately, for an 84 min pendulum with a radius of gyration of 1 in., the pivot-center of mass separation would have to be of the order of only a few Ångstrom units.

An 84 min period is very long compared with the periods of physical pendulums ordinarily encountered. In fact, it is improbable that a simple (concentrated-mass) pendulum or a distributed-mass pendulum could be constructed with this period, although the 84 min gyro-pendulum just fails being realizable. However, in the practical case, a condition

Derivation Summary 1. Geocentric accelerations.

Acceleration with respect to inertial space of a point, P, moving over the Earth is expressible in terms of components of

a. acceleration of P relative to the Earth

b. acceleration of the Earth relative to inertial space

Since Earth-centered inertial frame I and Earth frame E are concentric, the application of the law of Coriolis to the inertial acceleration$^{(17,18)}$ $[\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{I}}$ of point P is, in terms of acceleration relative to the Earth $[\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{E}}$ and the Earth's inertial motion

$$[\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{I}} = [\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{E}} + \bar{W}_{\mathrm{IE}} \times (\bar{W}_{\mathrm{IE}} \times \bar{R}_{\mathrm{EP}}) + 2\bar{W}_{\mathrm{IE}} \times [\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{E}} \tag{1-1}$$

where

$\bar{W}_{\mathrm{IE}}$ = daily sidereal rotation of the Earth

All terms in the right-hand side are referred to the Earth frame E.

A similar treatment of $[\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{E}}$ gives

$$[\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{E}} = [\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{C}} + \bar{W}_{\mathrm{EC}} \times (\bar{W}_{\mathrm{EC}} \times \bar{R}_{\mathrm{EP}}) + [\dot{\bar{W}}_{\mathrm{EC}}]_{\mathrm{E}} \times \bar{R}_{\mathrm{EP}} + 2\bar{W}_{\mathrm{EC}} \times [\dot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{C}} \tag{1-2}$$

All terms in the right-hand side are referred to geocentric position frame C.

Substitution of Eq. (1-2) into Eq. (1-1) gives

$$\begin{aligned}[\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{I}} &= \bar{W}_{\mathrm{IE}} \times (\bar{W}_{\mathrm{IE}} \times \bar{R}_{\mathrm{EP}}) + 2\bar{W}_{\mathrm{IE}} \times (\bar{W}_{\mathrm{EC}} \times \bar{R}_{\mathrm{EP}}) \\ &\quad + [\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{C}} + \bar{W}_{\mathrm{EC}} \times (\bar{W}_{\mathrm{EC}} \times \bar{R}_{\mathrm{EP}}) + [\dot{\bar{W}}_{\mathrm{EC}}]_{\mathrm{E}} \times \bar{R}_{\mathrm{EP}} + 2\,(\bar{W}_{\mathrm{IE}} + \bar{W}_{\mathrm{EC}}) \times [\dot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{C}}\end{aligned} \tag{1-3}$$

where

$\bar{W}_{\mathrm{IE}} \times (\bar{W}_{\mathrm{IE}} \times \bar{R}_{\mathrm{EP}})$ = daily Earth rotation centripetal acceleration, combined with gravitation $\bar{G}$ to make gravity $\bar{g} = \bar{G} - \bar{W}_{\mathrm{IE}} \times (\bar{W}_{\mathrm{IE}} \times \bar{R}_{\mathrm{EP}})$

$2\bar{W}_{\mathrm{IE}} \times (\bar{W}_{\mathrm{EC}} \times \bar{R}_{\mathrm{EP}})$ = Coriolis acceleration due to coupling of Earth rate $\bar{W}_{\mathrm{IE}}$ with ground speed $[\dot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{E}} = \bar{W}_{\mathrm{EC}} \times \bar{R}_{\mathrm{EP}}$; is to be compensated

$[\ddot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{C}}$ = radial acceleration; usually negligible

$\bar{W}_{\mathrm{EC}} \times (\bar{W}_{\mathrm{EC}} \times \bar{R}_{\mathrm{EP}})$ = centripetal acceleration due to ground speed; usually negligible

$[\dot{\bar{W}}_{\mathrm{EC}}]_{\mathrm{E}} \times \bar{R}_{\mathrm{EP}}$ = tangential acceleration due to angular acceleration $[\dot{\bar{W}}_{\mathrm{EC}}]_{\mathrm{E}}$ associated with linear acceleration; most important term

$2\,(\bar{W}_{\mathrm{IE}} + \bar{W}_{\mathrm{EC}}) \times [\dot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{C}}$ = Coriolis acceleration due to coupling of Earth rate $\bar{W}_{\mathrm{IE}}$ and ground speed, associated with angular velocity $\bar{W}_{\mathrm{EC}}$, with radial velocity $[\dot{\bar{R}}_{\mathrm{EP}}]_{\mathrm{C}}$; usually negligible

analogous to the Schuler tuning of a pendulum can be realized in a particular kind of electromechanical closed-loop system. A platform driven by a space integrator and controlled by signals derived from a short-period pendulum or accelerometer then becomes an equivalent Schuler pendulum. This configuration is illustrated in Fig. 4.

Closed-loop Equivalent Pendulums—Gravity Tracking

A Schuler pendulum can be achieved in practice by making the force-sensitive properties separate from the torque-producing properties, in two distinct sub-systems, and inserting a dynamic control function between them. The mechanical features of an equivalent Schuler pendulum are shown in Fig. 4, and its functional properties in Fig. 5.

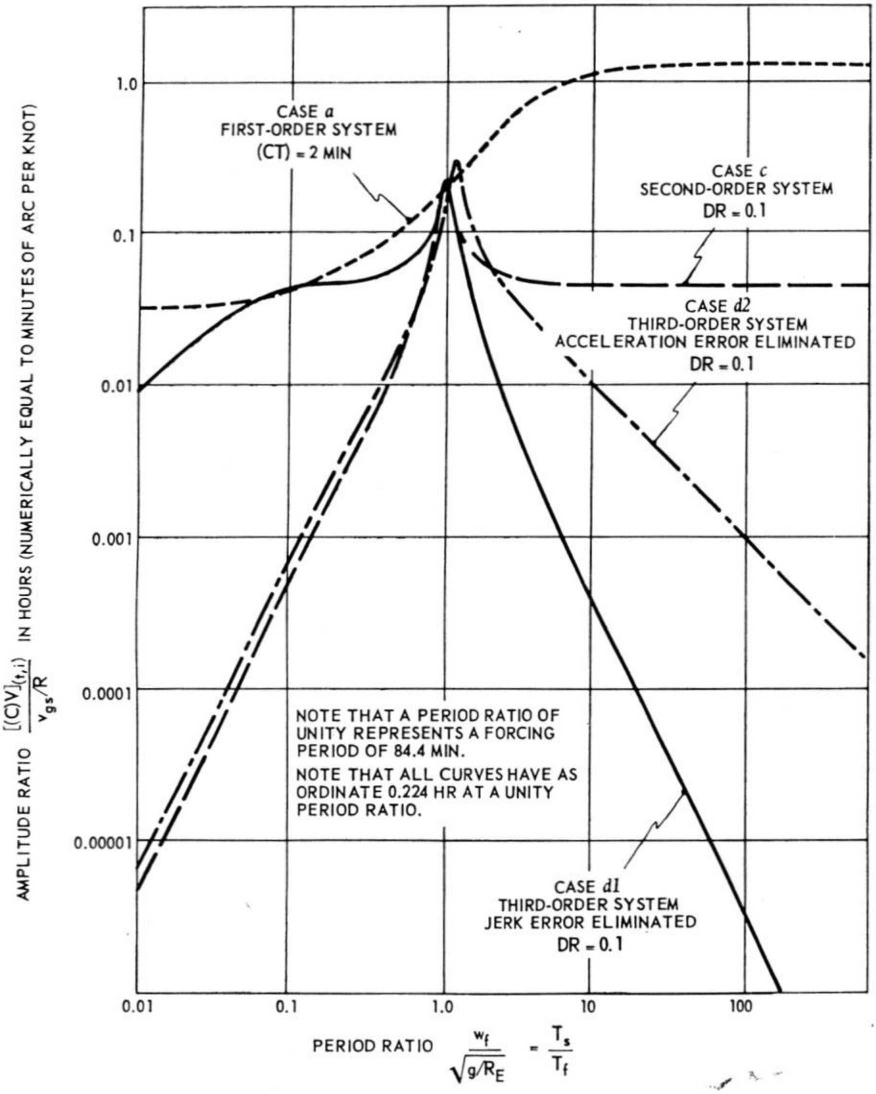
The specific force receiver indicates the resultant specific force acting on the vertical indicator. When the vertical indicator is accelerated over the Earth, this force is no longer aligned with gravity, which it is desired to track. The specific force is along the *apparent vertical*, while gravity is along the *true vertical*. Thus, vertical indication on an accelerated craft presents the peculiar problem of tracking gravity by means of a device that can actually track only the total specific force acting on the tracking element.

When the dynamic control is an integrator, the vertical indicating system performance is of second order and undamped, exhibiting forced dynamic acceleration error. This error is a function of the natural period of the pendulum. Schuler tuning involves proper control of this natural period.

Since the Schuler-tuned system thus described is undamped, it is subject to unending oscillations, which may possibly even increase in magnitude due to component imperfections. The necessary damping in a vertical indicator is obtained by modifying the dynamic control function.

In the simplest case, the dynamic control function is a direct coupling, and the performance is describable by a first-order differential equation. This is the artificial horizon, which acts like a heavily over-damped pendulum, and cannot be Schuler tuned. This vertical indicator is subject to both velocity and acceleration (of the craft) forced dynamic errors. The magnitude of the velocity error is determined by the strength of the dynamic control function; the stronger the coupling, the smaller the velocity error. The acceleration error is independent of coupling strength, once the forcing period exceeds a certain value, being approximately the ratio of craft acceleration to gravity. This is the same error displayed by any short-period pendulum, reflecting its tendency to track the apparent vertical rather than the true vertical. The artificial horizon is a good indicator of the vertical against roll, pitch and yaw, but becomes progressively poorer as the periods associated with craft maneuvers lengthen. Its performance, which for typical data is shown as Case (*a*) in Fig. 9, is not suitable for inertial guidance.

If a bypass is added around the coupling integrator, the loop is damped, and will settle to a steady state. However, the addition of the bypass, while producing damping, also results in a forced dynamic error. The gravity, i.e. tilt, component of the specific-force signal leads to the elastic term via the integrator and to damping via the bypass. The acceleration component leads to the inherent tracking aid via the integrator, but introduces an unwanted superfluous term via the bypass. In this case, the forced dynamic



NOTE THAT A PERIOD RATIO OF UNITY REPRESENTS A FORCING PERIOD OF 84.4 MIN.

NOTE THAT ALL CURVES HAVE AS ORDINATE 0.224 HR AT A UNITY PERIOD RATIO.

Fig. 9. Steady-state amplitude ratio – period ratio response for vertical indication loops.

error is an acceleration derivative, or jerk, error that tends to become predominant at frequencies high compared with the Schuler frequency. Forced errors due to damping may be minimized by injecting externally derived compensating inputs, with the effectiveness of the forced-error cancellation determined by the accuracy of compensation. This method is shown in Case (*c*) in Fig. 9.

The above-discussed integrator-with-bypass is sometimes called an *integration-with-lead network* or high-pass filter. This produces a "noisy" system, which can be markedly improved by the addition of a lag network, or low-pass filter. The immediate result of this more-elaborate coupling scheme is a choice of two Schuler-tuning methods: the Earth-radius can enter as a loop-gain parameter in either of two ways. In one case, the acceleration error will be eliminated and the analogy with the Schuler tuning of a physical pendulum is strong; in the second case, the jerk error is eliminated. This method is shown as Cases (*d*1) and (*d*2) in Fig. 9, where it is seen that Case (*d*1) gives the best performance. It should be noted that a Schuler-tuned vertical indicator always operates well into its high-frequency regime, due to the long natural period associated with Schuler tuning.

Case (*d*1) of Fig. 9, which gives the best performance, is analogous to the "differentiated-tachometer-feedback" type of damping found to be so effective with servo-mechanisms.

Schuler tuning in an equivalent pendulum can be derived from two points of view, presented in Derivation Summary 2. From the first viewpoint, the Schuler-tuning adjustment consists of adjusting a *calibrated dynamic lag* into the system. From this viewpoint, the indicated vertical dynamically lags the apparent vertical, which in turn geometrically leads the true vertical. The apparent vertical is parallel to the direction the true vertical will have at some time in the near future; hence the term "lead". Figure 8 illustrates this geometry. By Schuler calibration, the dynamic lag can be matched to the geometrical lead. This means the indicated vertical will effectively follow the true vertical. From the second point of view, a *calibrated tracking aid* is made an inherent part of the system when it is Schuler tuned. From this viewpoint, the acceleration input is an inherent tracking-aid lead, which can be made to compensate the dynamic lag that otherwise would exist between indicated and true verticals. Schuler tuning involves proper use of this inherent lead factor. These approaches are, of course, but two different views of a single problem, that of rendering tangential accelerations of the vertical indicator on an assumed spherical Earth helpful and necessary rather than disturbing. The assumption of a spherical Earth has been used for simplicity of presentation, and does not restrict the validity of Schuler tuning relative to the actual Earth. In summation, gravity represents a direction and acceleration represents the second angular rate of change of that direction. They are inseparable dynamically as well as physically, and are properly co-ordinated by Schuler tuning.

AZIMUTH INDICATION

The local co-ordinate frame in an inertial guidance system is determined primarily by the direction of gravity. In order to make a complete determination of this frame it is also necessary that rotations about the gravity

Derivation Summary 2. The Schuler tuning of a gravity tracker.

Schuler Tuning as a Calibrated Dynamic Lag

To show how Schuler tuning corresponds in its effects to a calibrated dynamic lag in a vertical indicator, first define the *correction to the indicated vertical with respect to the apparent vertical* $[\overline{(C)V}]_{(a,i)}$ as the rotation that would have to be applied to the indicated vertical to align it with the apparent vertical. Then

$$[\overline{(C)V}]_{(a,i)} \equiv \bar{I}_{Vi} \times \bar{I}_{Va}\, q_{(a,i)} \qquad (2\text{-}1)$$

where

$\bar{I}_{Vi} \equiv$ unit vector along the indicated vertical

$\bar{I}_{Va} \equiv$ unit vector along the apparent vertical ; $q_{(a,i)} \equiv \dfrac{[(C)V]_{(a,i)}}{\sin\,[(C)V]_{(a,i)}}$

This is a *vector angle*; its rate of change with respect to the Earth is[*]

$$\{p\,[\overline{(C)V}]_{(a,i)}\}_E = \bar{W}_{(EV)a} - \bar{W}_{(EV)i} \qquad (2\text{-}2)$$

where

$\bar{W}_{(EV)a} \equiv$ angular velocity of the apparent vertical relative to the Earth

$\bar{W}_{(EV)i} \equiv$ angular velocity of the indicated vertical relative to the Earth

From the vertical indicator loop, Fig. 2-1,[**]

$$\bar{W}_{(EV)i} = S_{(Vi)[f;\dot{W}]}\,\frac{1}{p}\,(\bar{I}_{Vi} \times \bar{f}) + \bar{W}_{[(EV)i]_o} \qquad (2\text{-}3)$$

where

$S_{(Vi)[f;\dot{W}]} = S_{(sfr)[f;e]}\,S_{(int)[e;i]}\,S_{(si)[i;W]}$
$\quad = $ the vertical indicator sensitivity, the product of specific force receiver, integrator and drive sensitivities

$S_{(sfr)[f;e]} =$ specific force receiver sensitivity

$S_{(int)[e;i]} =$ signal integrator sensitivity

$S_{(si)[i;W]} =$ space integrator sensitivity

$\bar{f}$ is the total specific force along the apparent vertical; thus

$$\bar{I}_{Va} \equiv \frac{\bar{f}}{f} \qquad (2\text{-}4)$$

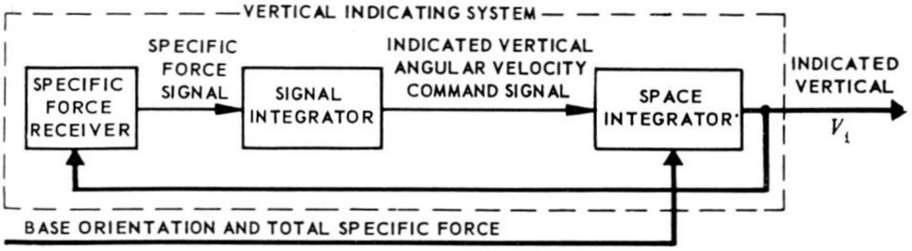


Fig. 2-1. Single-axis vertical indicating loop.

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[*] Assuming that no rotation *about* the indicated vertical occurs, i.e., an azimuth control or compass operates to stabilize the vertical indicator about the vertical.

[**] Note that while the accelerometer actually responds to $[(\bar{I}_{Vi} \times \bar{f}) \times \bar{I}_{Vi}]$, the drive is so oriented relative to the accelerometer that *it* responds to a function of $\bar{I}_{Vi} \times [(\bar{I}_{Vi} \times \bar{f}) \times \bar{I}_{Vi}]$ which equals $(\bar{I}_{Vi} \times \bar{f})$, as given in the equation.

Derivation Summary 2. The Schuler tuning of a gravity tracker.

Note that

$$\overline{I}_{\mathrm{Vi}} \times \overline{f} = \overline{I}_{\mathrm{Vi}} \times \overline{I}_{\mathrm{Va}} f = \frac{f\left[\overline{(C)V}\right]_{(a,i)}}{q_{(a,i)}} \tag{2-5}$$

so that from the foregoing,

$$\left[p^2 + \frac{S_{(\mathrm{Vi})[\mathrm{f};\dot{\mathrm{W}}]} f}{q_{(a,i)}}\right]\left[\overline{(C)V}\right]_{(a,i)} = p\,\overline{W}_{(\mathrm{EV})a} \tag{2-6}$$

This is the equation of motion for a system tracking the apparent vertical. Since the true vertical is desired, it is necessary to investigate the geometrical relationships between the apparent and true verticals. It is subsequently shown that, in the presence of accelerations, the apparent vertical leads the true vertical, i.e., its direction is parallel to the direction the true vertical will attain in the near future. The problem is to make the indicated vertical lag the apparent vertical by the same angle that the apparent vertical leads the true vertical. Define this latter angle $\overline{A}_{(t-a)}$ as

$$\overline{A}_{(t-a)} = \overline{I}_{\mathrm{Vt}} \times \overline{I}_{\mathrm{Va}}\, q_{(t,a)} \tag{2-7}$$

and

$$\left[p\overline{A}_{(t-a)}\right]_{\mathrm{E}} = \overline{W}_{(\mathrm{EV})a} - \overline{W}_{(\mathrm{EV})t} \tag{2-8}$$

where

$$\overline{I}_{\mathrm{Vt}} \equiv \frac{\overline{g}}{g} \cong -\frac{\overline{R}_{\mathrm{E}}}{R_{\mathrm{E}}}$$

$$q_{(t,a)} \equiv \frac{A_{(t-a)}}{\sin A_{(t-a)}}$$

$\overline{W}_{(\mathrm{EV})t}$ ≡ angular velocity of true vertical relative to the Earth

$\overline{g}$ ≡ gravity

$\overline{R}_{\mathrm{E}}$ ≡ the (assumed constant magnitude) Earth-radius

The tangential acceleration of the vertical indicator* on the Earth is considered to be

$$\overline{a} = p\,\overline{W}_{(\mathrm{EV})t} \times \overline{R}_{\mathrm{E}} \tag{2-9}$$

so that

$$\overline{f} = \overline{g} - \overline{a} = \overline{I}_{\mathrm{Vt}}\, g - p\,\overline{W}_{(\mathrm{EV})t} \times \overline{R}_{\mathrm{E}} \tag{2-10}$$

Note that the inertia reaction effect of an acceleration is opposite in direction to the acceleration. The cross product of $\overline{I}_{\mathrm{Vt}}$ into Eq. (2-10) gives $\overline{I}_{\mathrm{Vt}} \times \overline{f} = R_{\mathrm{E}}\, p\,\overline{W}_{(\mathrm{EV})t}$. Substitution of this into the derivative of Eq. (2-8) and rearrangement of terms gives

$$p\,\overline{W}_{(\mathrm{EV})a} = \left[p^2 + \frac{f}{R_{\mathrm{E}}\, q_{(t,a)}}\right]\overline{A}_{(t-a)} \tag{2-11}$$

If the correction to the indicated vertical relative to the apparent vertical, or angle by which the indicated vertical lags the apparent vertical, is to equal the angle by which the apparent vertical leads the true vertical, i.e., if

$$\left[\overline{(C)V}\right]_{(a,i)} = \overline{A}_{(t-a)} \tag{2-12}$$

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* For the assumed condition of a spherical nonrotating Earth (which simplifies the problem for demonstrating the Schuler effect without restricting its validity) the tangential acceleration is the only important acceleration term. The full equation is Eq. (1-3) in Derivation Summary 1.

Derivation Summary 2. The Schuler tuning of a gravity tracker.

it is necessary that, comparing Eqs. (2-6) and (2-11),

$$S_{(\mathrm{Vi})[\mathrm{f};\dot{\mathrm{w}}]} \equiv \frac{1}{R_s} = \frac{1}{R_E} \qquad (2\text{-}13)$$

which is the Schuler tuning condition, where R_s is the Schuler radius.

Schuler Tuning as a Calibrated Tracking Aid

Schuler tuning can also be equated to the calibration of an inherent acceleration lead signal due to angular acceleration of the vertical indicator over an essentially spherical Earth. This angular acceleration is actually then made useful in overcoming the lag of the apparent vertical behind the true vertical during accelerations. To see how this comes about, define

$$[\overline{/(C)V}]_{(t,i)} \equiv \bar{I}_{\mathrm{Vi}} \times \bar{I}_{\mathrm{Vt}} \qquad (2\text{-}14)$$

where $q_{(t,i)}$ is practically unity, since $[/(C)V]_{(t,i)}$ is a small angle. Note that (prohibiting rotation of the indicated about the true vertical by an ideal azimuth stabilizer, as before)

$$\{p\,[\overline{/(C)V}]_{(t,i)}\}_E = \overline{W}_{(\mathrm{EV})t} - \overline{W}_{(\mathrm{EV})i} \qquad (2\text{-}15)$$

while from Fig. 2-1, $\overline{W}_{(\mathrm{EV})i}$ is, as before, given by

$$\overline{W}_{(\mathrm{EV})i} = S_{(\mathrm{Vi})[\mathrm{f};\dot{\mathrm{w}}]}\,\frac{1}{p}\,(\bar{I}_{\mathrm{Vi}} \times \bar{f}) + \overline{W}_{[(\mathrm{EV})i]_o} \qquad (2\text{-}16)$$

From the foregoing, including Eq. (2-10),

$$[p^2 + S_{(\mathrm{Vi})[\mathrm{a};\dot{\mathrm{w}}]}\,g]\,[\overline{/(C)V}]_{(t,i)} = [\underset{\substack{\uparrow \\ \text{lag}}}{1} - \underset{\substack{\uparrow \\ \text{calibrated} \\ \text{(tracking aid)} \\ \text{lead}}}{S_{(\mathrm{Vi})[\mathrm{f};\dot{\mathrm{w}}]}}\,R_E]\,p\,\overline{W}_{(\mathrm{EV})t} \qquad (2\text{-}17)$$

When the condition

$$S_{(\mathrm{Vi})[\mathrm{f};\dot{\mathrm{w}}]} \equiv \frac{1}{R_s} = \frac{1}{R_E} \qquad (2\text{-}18)$$

is applied, the equation of motion of the vertical indicator is*

$$\left[p^2 + \frac{g}{R_s}\right]\,[\overline{/(C)V}]_{(t,i)} = 0 \qquad (2\text{-}19)$$

and the vertical indicator is Schuler tuned. This means that the second term in the right-hand side of Eq. (2-17), which represents calibration of the inherent acceleration lead, balances the first term that represents dynamic lag. This is a form of acceleration *tracking aid*.

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* Strictly speaking, g should be replaced by the net vertical specific force: $f_V = g - a_V$.

vector be known. This process is known as *azimuth* or *heading* indication. Azimuth indication furnishes two basic data for guidance:

(a) Direction in the horizontal plane from the local point to its destination, or any other reference position.

(b) Orientation of the vertical indicating system such that the components of position can be measured in terms of the navigational parameters desired, i.e. latitude–longitude, great circle, etc. All guidance systems require some form of heading information.

INERTIAL GUIDANCE INDICATING SYSTEMS

The vertical indicating system furnishes local information needed for inertial guidance; the other, equally important, part is the inertial reference. Three basic methods of coupling between these two sub-systems to obtain position data are available:

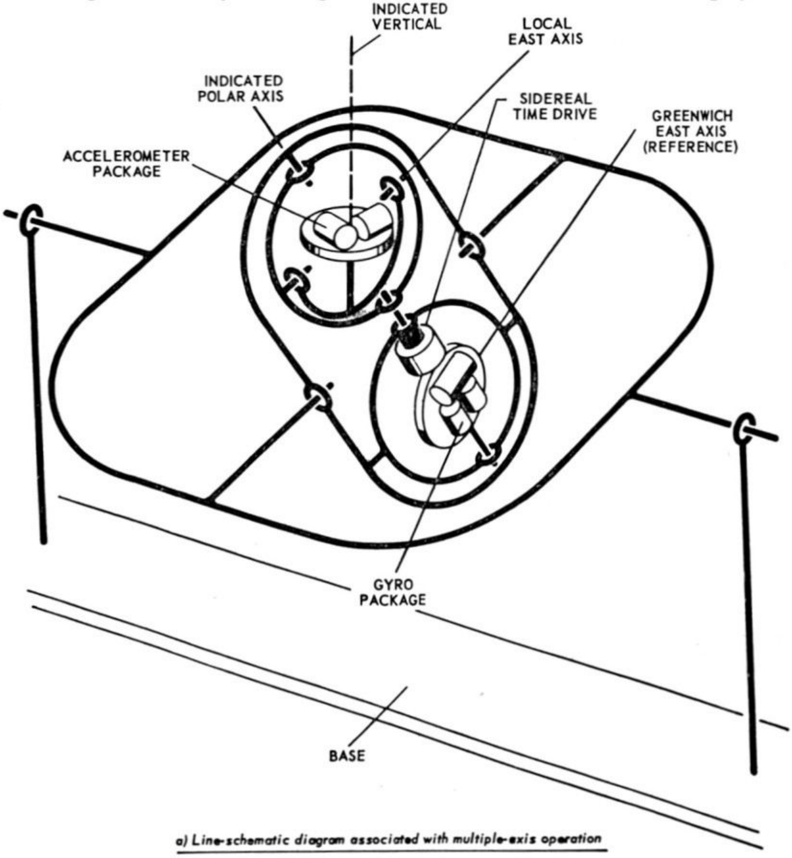
(a) Direct angular measure of position between the indicated vertical and reference verticals, the latter maintained directly by the gyro package.

(b) Integration of measured ground speed from the input to the space integrator drives of the vertical indicator.

(c) Double integration of the measured acceleration output of the accelerometers.

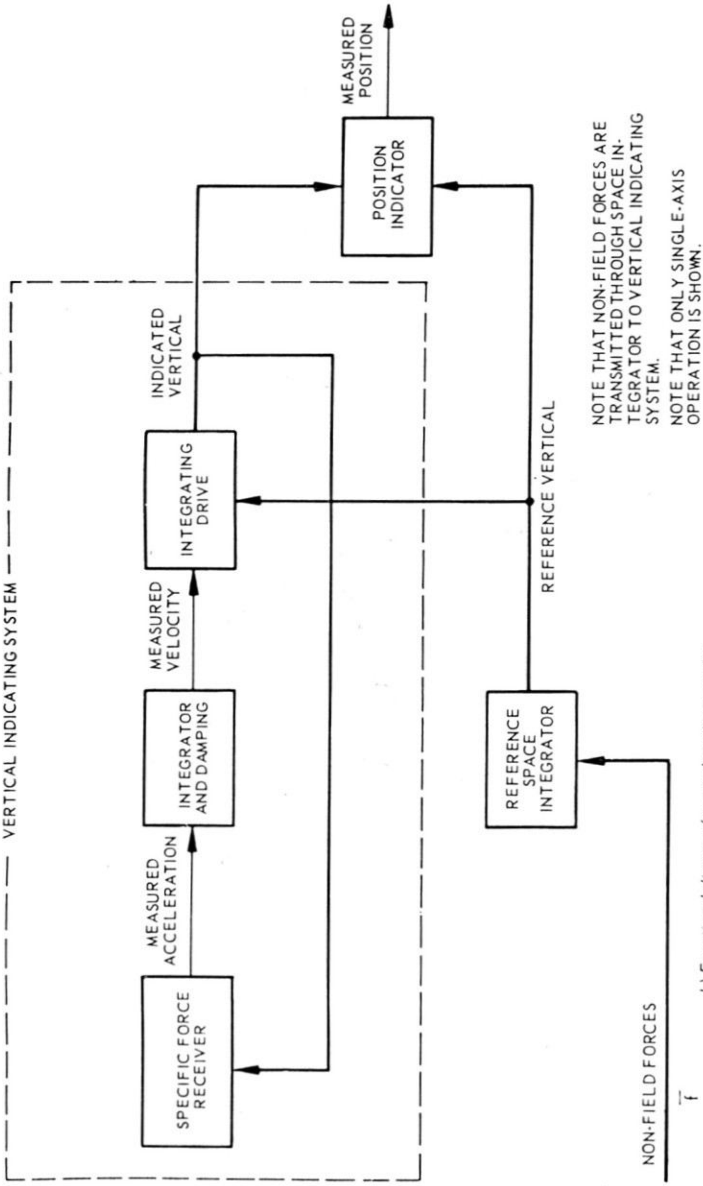
Direct Angular Measurement

This method is illustrated geometrically in Fig. 10a and functionally, for a single axis only, in Fig. 10b. The inertial reference is physically



a) Line-schematic diagram associated with multiple-axis operation

Fig. 10. The use of a physically-represented reference vertical aboard the vehicle to indicate position.



NOTE THAT NON-FIELD FORCES ARE TRANSMITTED THROUGH SPACE INTEGRATOR TO VERTICAL INDICATING SYSTEM.

NOTE THAT ONLY SINGLE-AXIS OPERATION IS SHOWN.

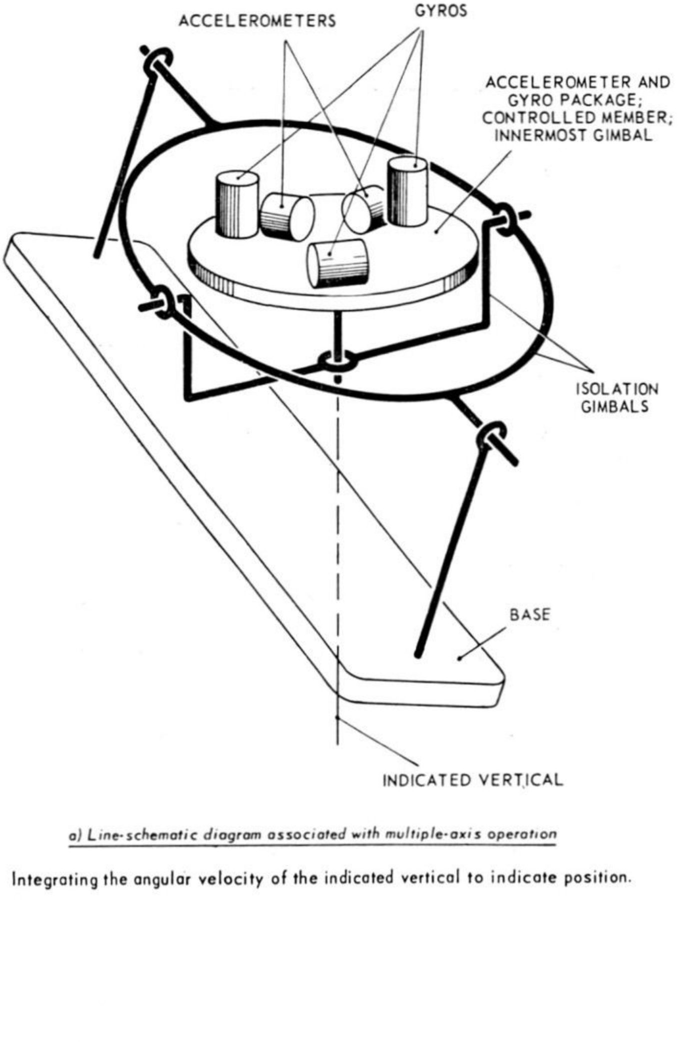
b) Functional diagram for single-axis operation

Fig. 10. The use of a physically-represented reference vertical aboard the vehicle to indicate position.

maintained by the space integrator, that is, by the gyro package and its associated drives. The gyros receive no commands, and so are a direct representation of inertial, or "fixed-star", space. The transition from inertial space to the Earth reference space is via the innermost gimbal drive, whose axis is then made as nearly parallel as possible to the Earth's polar axis. This drive is a synchronous clock operating from sidereal time. The indicated vertical is, by virtue of the vertical indicating system, aligned with the true vertical within a dynamic error zone, because the true vertical is the direction of gravity, which is a tracked quantity. The integrators operate within closed loops, so their drift affects only the loop dynamics. Only the gyro drifts are of an open-chain nature, and so could possibly produce position inaccuracies that might increase with increasing time of operation. Systems of this nature are best suited for high accuracies over long periods of time.

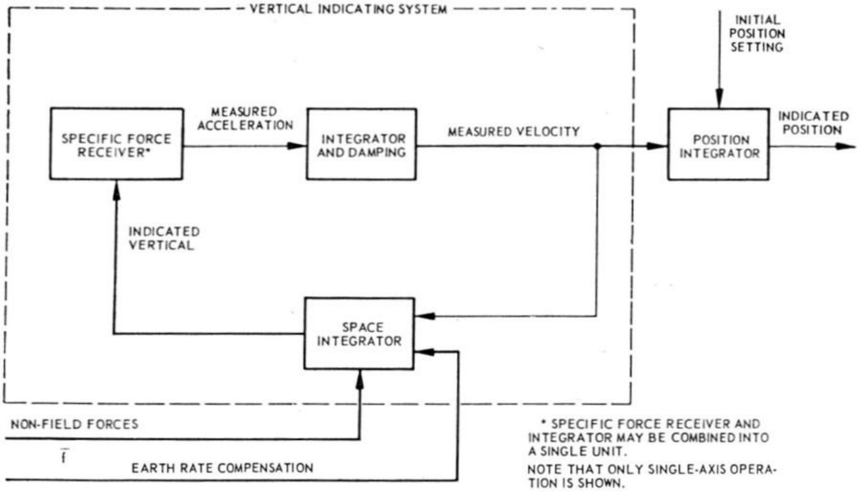
Integration of Measured Angular Velocity

This method is illustrated geometrically in Fig. 11a and functionally, for



a) Line-schematic diagram associated with multiple-axis operation

Fig. 11. Integrating the angular velocity of the indicated vertical to indicate position.



b) Functional diagram for single-axis operation

Fig. 11. Integrating the angular velocity of the indicated vertical to indicate position.

a single axis only, in Fig. 11b, where the vertical indicating loop in Fig. 2-1 of Derivation Summary 2 has been "bent" to show the space-integrator input as the desired output for this situation. In this case, the specific force receiver package is mounted rigidly with the gyro package. The space-integrator action ensures that the angular velocity of the indicated vertical relative to inertial space will be proportional to the total input signal to the space integrator; or, conversely, that the input signal will be a measure of the ground speed of the craft—when corrected for the signal component required to keep the vertical up to the daily rotation of the Earth. Position measurement is subject to drifts due to position-integrator drift, inaccuracy in applying the required Earth-rate component signal to the space integrator, and to gyro drift. Such systems are most suited to small-space and short-operating-time requirements, and where lower accuracy is allowable. In this system, the inertial reference appears as a constant of integration, i.e. a signal, due to the gyro's precession relative to inertial space under the action of a command. The physical inertial reference is "left behind", not carried with the system.

Double Integration of Measured Acceleration

This method is illustrated geometrically in Fig. 11a and functionally, for a single axis only, in Fig. 12, where the vertical indicating loop in Fig. 2-1 of Derivation Summary 2 has been "bent" to show the specific force receiver output as the desired output for this situation. It is geometrically identical to the measured-ground-speed system above. It

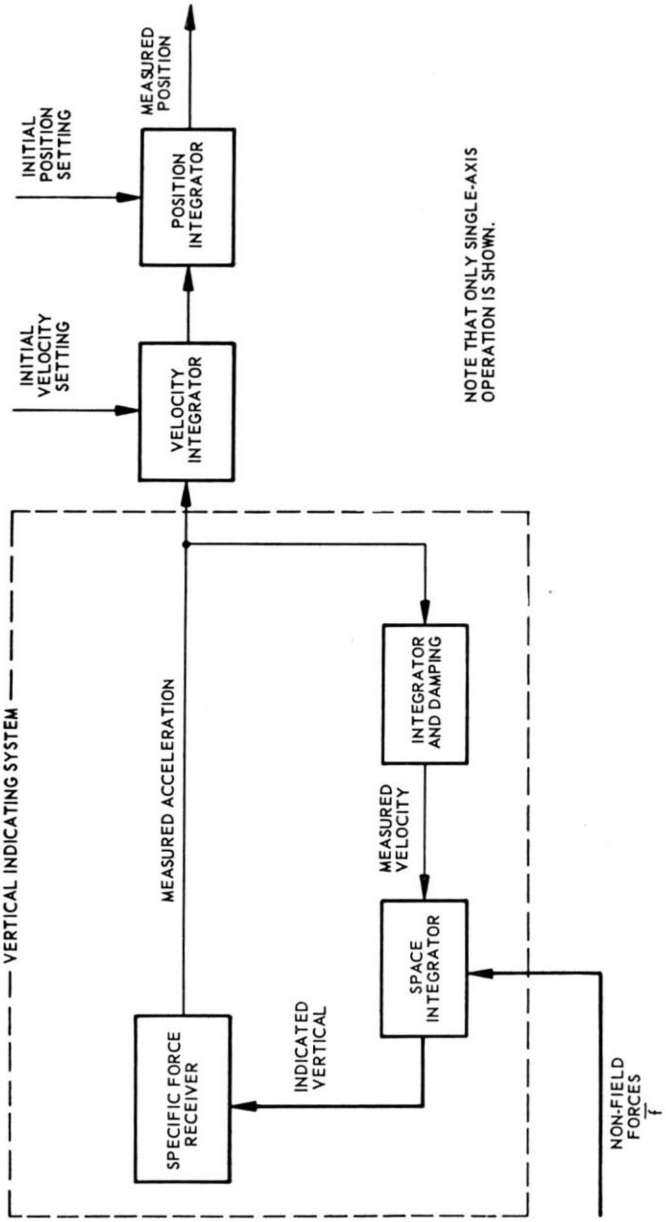
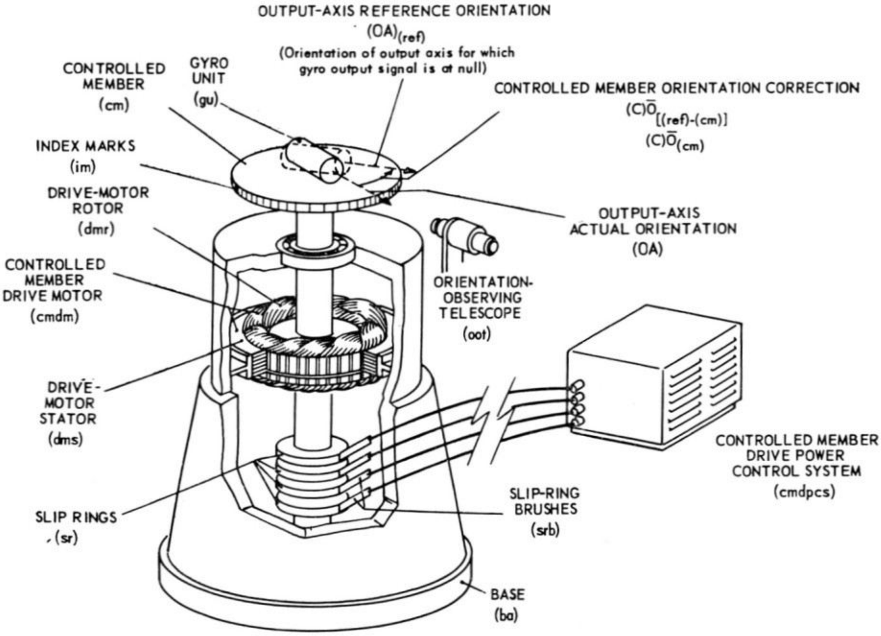
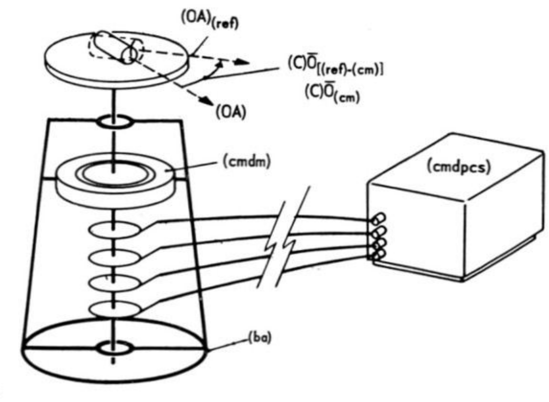


Fig. 12. Doubly-integrating the angular acceleration of the indicated vertical to indicate position.

further suffers from the cumulative double-integration drifts, which can introduce position errors that may be parabolic in time. This is dynamically the least suitable type of inertial indicating system.



a) Elementary pictorial diagram of single-axis servo-table gyro test equipment



b) Line schematic of single-axis servo-table gyro test equipment

Fig. 13. Pictorial diagram showing the elements of an illustrative single-axis controlled member inertial space geometrical stabilization system.

SPACE-INTEGRATOR PERFORMANCE

Space integration is the process of receiving signal inputs and producing outputs in the form of corresponding angular velocity components with respect to inertial space. The effective performance of this function is possible only if the system involved is insensitive to electrical and mechanical disturbing effects. Electrical interference is reduced to tolerable levels by the usual techniques of design, shielding, and installation. Mechanical interference may be due to external torque components acting directly on the controlled member or to base oscillations that tend to move this member by inertial coupling and support bearing torques that act against motion between the base and the control member. Only these bearing torques are present under static conditions, while both bearing torque and inertial coupling occur when the base is rotating. The essential features of the space integrator control system problem are illustrated by the pictorial and line schematic diagrams of Fig. 13. This figure actually represents a test table for single-degree-of-freedom gyro units, with a servodrive supplied with power from an electronic amplifier on the basis of signals from the gyro unit under test.

Figure 14 is a functional diagram for the gyro-test-table system. The base supports the controlled member shaft, which also carries the rotor of the drive motor. The gyro unit is fixed to the controlled member, with its input axis parallel to the axis of rotation of the controlled member shaft. The gyro unit is shown as receiving the orientation of the controlled member, the input command signal and the modifying inputs that correspond to the excitations and environmental control quantities necessary to keep the unit operating properly. The output signal from the gyro unit, which represents the angular correction to the orientation of the controlled member required to bring the actual orientation of the output axis into coincidence with the reference orientation of the output axis, is the input for the *controlled member drive motor power control system*. This system consists of a number of components including (a) the a.c. preamplifier, (b) the demodulator, (c) the signal modifier, (d) the d.c. amplifier, (e) the remodulator, (f) the a.c. post-modulator amplifier, and (g) the drive motor power amplifier. The output current from this latter amplifier is the input for the controlled member drive motor and interacts with the drive motor excitation to apply torque to the controlled member.

In practice, it is possible to achieve interference-torque input—angular-motion output sensitivities less than two hundredths of a milliradian per foot pound.

Figure 15 shows typical amplitude-ratio response curves for practical space-integrator systems in terms of the equivalent undamped natural frequency of the controlled member geometrical stabilization system. It should be noted that:

1. The command-angle-input response holds near unity, its ideal value,

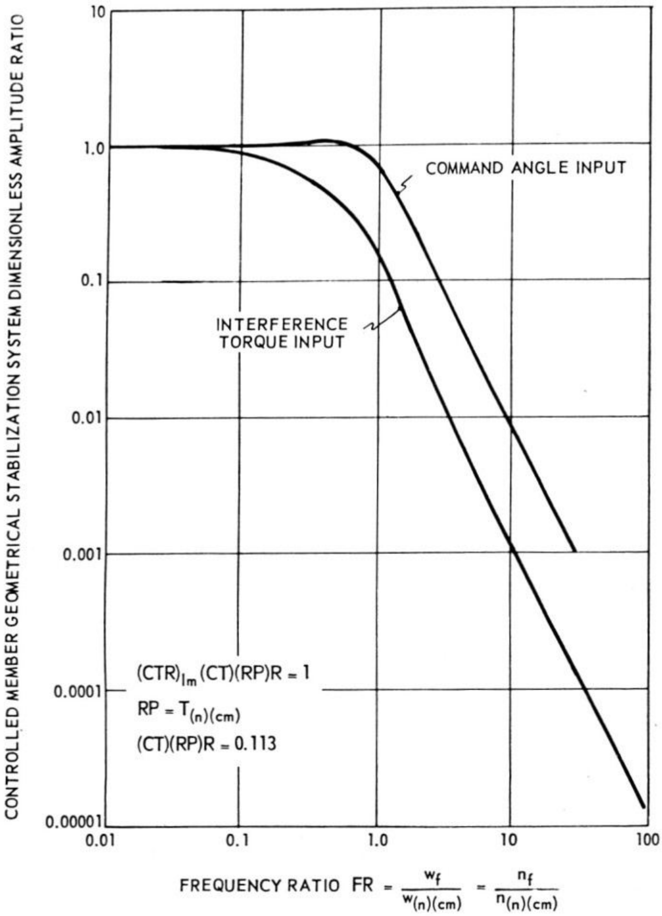


Fig. 15. Magnitude of frequency functions for a typical controlled member geometrical stabilization system.

to a higher frequency than the frequency at which the interference response begins to drop off from its low-frequency level.

2. In any practical system, the reference level for this low-frequency response is several orders of magnitude down from unity. In order to simplify the plot, this reference is not shown at its actual position.

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